14. Standing Waves and Resonance

Sound and other wave phenomena are associated with traveling waves in which a disturbance moves through some medium. The material itself jiggles back and forth a tiny distance, but the wave pattern moves over long distances. The wave transports energy over long distances, even though molecules in the medium moves only microscopic distances.

Learning objectives:
1. Learn that standing waves can be seen as the sum of two traveling waves.
2. Understand how standing waves occur at certain resonance frequencies, due to boundary conditions, such that the length of the object relates to the wavelength.

Your group works for the Gallasini Company that has been making musical instruments since the eighteenth century. In particular you are assigned work on resonance of sound waves in organ pipes and on resonant modes of a vibrating string.

Reading: Before lab, read the sections on wave motion, traveling waves, standing waves and resonance.
(161): Ch. 16 & 17, (211): Ch. 15 & 16, (252): Ch. 16 & 17.
Knight, Jones & Field (161): Chapters 15 & 16
Serway and Vuille (211): 13.1 through 13.7
Serway and Jewett (251): Chapter 16, 17.1 through 17.3

Vocabulary and recap: Here are reminders of vocabulary and basic concepts. The figure shows four time frames, time increasing top to bottom. It represents a traveling wave. A sound wave is traveling to the right. Grey bars represent compression of air in the tube, so where they are closer together the air is compressed and where they are further apart is a slight vacuum. The air molecules don’t move much but the pattern moves. The wavelength $\lambda$ is the length of one cycle of the wave, so it is the distance between two regions of maximum compression. If you stay at one point and watch the wave go by you observe pressure going up and down in time, the time for one cycle is the period $\tau$ of the wave. So $\lambda$ is in meters and $\tau$ is in seconds. The wave moves along at speed $v$ that is the speed of sound in a given situation. Thus we have the very fundamental relation $v = \lambda / \tau = \text{distance per cycle} / \text{time per cycle}$. 

PRE-LAB EXERCISES

Before coming to lab, work out the following. Your TA will check that you have done the exercises when you come to lab. Later you will turn them in as part of your report.

1. Consider first pressure versus position at fixed time for a traveling sound wave. On the upper axes, sketch the over-pressure (pressure minus atmospheric pressure) as vertical and position along the wave as horizontal for a sound wave traveling toward the left. Make the lower graph at a slightly later time than the upper graph, but for the same sound wave. On each graph, add an arrow to indicate the distance of one wavelength. Label the axes.

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2. Consider again the same wave traveling toward the left, but this time graph pressure versus time at fixed position for the sound wave. Sketch the over-pressure as vertical and time as horizontal. Make the lower graph at a position slightly to the right of that in the upper graph, but for the same wave. On each graph, add an arrow to indicate the time interval corresponding to one period. Label the axes.

3. So much for traveling waves, now let’s review standing waves. When a traveling wave runs into a barrier—like a wall in the case of sound—it is reflected. The reflected wave has the same wavelength and frequency, but moves in the opposite direction. But, **what happens is that the original wave and the reflected wave add up** to produce a standing wave. The standing wave doesn’t move to the right or left, but jiggles up and down in a “jump-rope” like pattern. In the four frames at the right you see two traveling waves, one going right (solid) and one going left (dashed). The overpressures add at each instant to form the standing wave. It’s your job to take your pencil and sketch in the sum for each of the three sets of curves.
4. On the set of waves above locate at least two nodes, or places where the total standing-wave amplitude remains zero at all times.

5. On the axes below, construct (don’t just sketch) six different pictures of the standing wave you constructed in 3 and 4, each one at a different time during the cycle, but use the same origin for the horizontal axis, which is position in space. The result should look like a sort of jump-rope pattern, with the nodes (fixed points) standing out plainly.

6. At the right is a pictorial representation of a standing wave at four different, consecutive times. A node (the word node comes from knot) in the jump-rope picture you made is a point where the pressure does not fluctuate. Draw in at least two vertical lines on the picture to indicate where nodes would be. Think carefully.

7. Imagine an ant crawling along inside the tube from left to right. Describe in a sentence or two (use complete sentences) what the ant would see and how this correlates with where the ant is in the standing wave pattern as it crawls along.

**Equipment:** The lab problems relate to two types of instruments: wind instruments and stringed instruments. To study open-ended pipes, such as organ pipes, you have a tube that can be filled partially with water from a reservoir. By moving the reservoir up or down carefully you can adjust the water level in the tube. You also have a meter stick, tuning forks with a striking hammer for the tuning forks, as well as a supply of rubber bands. The rubber bands can be put around the tube in order to mark where the water level is for future reference.

For the second problem you will use a Mechanical Wave Driver (PASCO SF9324) and a Wave Generator (PASCO WA-9867). You should find about 3 meters of elastic cord, a bench-mounting pulley, an aluminum rod, a mass set with weight hanger and a two-meter stick. In the room there will also be a triple beam balance and weight set for determining mass.
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Problem 1

Resonance condition in an open-end pipe. A standing wave in a pipe with one end open and the other closed must have a velocity node at the closed end. A velocity node is where the molecules do not jiggle sideways, on average, so the pressure swing is maximum. (Why must the closed end be a velocity node?) At resonance, the wave pattern for a certain characteristic frequency fits nicely into the pipe and the sound is enhanced. Use the equipment to detect resonance at fixed frequency for at least two different lengths of a pipe with one end blocked at which there is resonance.

What is the difference (in meters) between two consecutive resonance lengths?

How can you explain this in terms of the nature of the standing waves?

Messing with the equipment: Explore the apparatus. Learn how to control the effective length of the plastic pipe by moving the water level. The group needs to figure out the best way to detect when there is resonance. If you start with water near the top, put a vibrating fork near the mouth of the tube, and then let the level go down, you should hear a distinct “wow” sound as the length passes through a resonance. To hear another resonance with the same tuning fork you have to listen very carefully because it’s a lot weaker. It will happen with the level quite a bit further down. Fiddle with different tuning forks and get good at finding resonances.

1. Method question: When you strike the tuning fork you should be using the rubber striker. There are two reasons. One is so as not to damage the fork. But the more important one has to do with controlling frequency. Bang the fork hard and notice the pitch of the initial “ping” sound. Then let the fork vibrate for two or more seconds and bring it up to your ear. Everyone should do this. How does the pitch of the pure tone you eventually hear differ from that of the initial ping? How can this effect screw up your experiment? Why is it better to use the rubber striker than to bang the fork on the table?

2. Method question: To detect resonance you have to hear a difference in sound volume as the length passes through a resonant length. So you need to detect a “wow”. What does this tell you about the way you need to make measurements? What should the water level be doing to make a distinct wow? Why does this complicate the length measurements?
3. Prediction question: Because the sound wave is reflected, the water surface should create a node in the standing wave. At the other end, so that sound gets in and out, there should be a velocity anti-node (more or less) where a velocity anti-node is a maximum in the velocity fluctuation and minimum in pressure fluctuations. (Actually it’s not quite an anti-node, but close.) Thus, if the speed of sound in open air is something like 350 m/s, what do you predict would be the length difference between two consecutive resonance conditions in the tube when the frequency is fixed by at tuning fork at 440 Hz? Explain, including a figure showing two jump-rope patterns in the pressure fluctuations.

**Plan:** Decide on a plan, based on the tuning forks and other equipment you have, and record your plan here.

**Implementation:** Make measurements and record the data, being careful as always to record the units and make notes to help you remember what you’ve got.

**Analysis:** Write out your prediction calculation and your reasoning, and then analyze your data to see whether there is agreement and to what extent.
Conclusions: So, what did you find out?
1. To what extent did your experiments support the prediction you made regarding the difference between resonance lengths?

2. What are the main problems with this type of experimental measurement?

Problem 2
(The standing-wave ratio) The Galasini Company needs to know about the end corrections to the resonance length. The theory is that the anti-node at the open end is not exactly at the end of the tube, but a bit further out in space, beyond the end. Thus the position of the first resonance should be a little different than it would be if there were an anti-node at the end.

Determine the position of the anti-node outside the end of the tube and whether this correction of the first resonance position depends on frequency or only on the tube diameter.

1. Method question: What have we been assuming about the velocity of sound inside the tube that might be significant here? Explain how it might be relevant.
2. Method question: Since we cannot change the diameter of the tube in this experiment, what length can we change, and how can this be related to changing the tube diameter? (Hint: Lengths can have no absolute meaning; they only have meaning compared to other lengths.)

3. Prediction question: Guess the distance for the first resonance measured inward from the end of tube to the water surface. Comment on what went into the group discussion to arrive at this guess.

Plan: Work out a measurement plan, including a prediction of what you expect to find and what you need to measure in order to see if you are correct. What quantities will you vary and what data will you need to get? It will be useful to design a data table. Record here.

Implementation: Take your data and record.

Analysis: Perform the analysis to see whether your predictions were reasonable. Show the analysis here, including explanation as appropriate.
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Conclusions:
What was learned?
1. Do you have a formula for the end correction based on fundamental physics? If so explain. In not, what things do you not know how to take into account?
2. Do you feel you have an estimate of the end correction for tubes of different diameters? What is it? To what extent can you estimate, based on your work, how this might depend on frequency of the source? Explain.

Now we switch from wind instruments to stringed instruments: Traveling waves in a stretched string are transverse, which means that the actual motion of each tiny part of the string is perpendicular to the string. As with sound waves or any waves, the wave speed, wavelength and frequency are related simply by the formula worked out above. The speed in this case is not the speed of sound. Rather it depends on properties of the string, and indeed on how tightly the string is stretched.

Problem 3

Here you will use dimensional analysis to work out a candidate formula for the speed of a traveling wave in a stretched string, and then perform measurements to see if your formula is consistent with measurements.

1. Prediction question: The figure above shows a traveling wave in a string. The dashed curve represents the same string as the solid curve, but just an instant earlier. Which direction is the wave traveling, to the right or to the left? Are you making any assumptions when you answer this?
2. Prediction question: Given that the wave speed should depend on properties of the string, such as tension $F$ or mass per unit length $\mu$, work out a formula for the wave speed by means of dimensional analysis alone. (No other input is to be used.) Explain your reasoning. Also, by thinking of what happens as you vary the input quantities, making them larger or smaller, discuss the reasonableness of your prediction.

3. Prediction question: Suppose you have a string of length $L = 2$ meters anchored at both ends and when you drive it at 30 Hz it is at resonance, moving in three segments with large amplitude. What is the speed of sound in the string under these conditions?

4. Method question: Given materials and instruments available, how can you make a relatively accurate measurement of the mass per unit length of the string?

5. Method question: If you attach one end of the string to a pole and hang masses on the other end after passing it over a pulley, how should the amount of suspended mass relate to the tension in the cord? Explain.

To make measurements, you may find it handy to set up your equipment more or less like this:

Your group needs to work with the physical equipment and the practice adjusting the frequency to understand how the driver works. It will be handy to have a long segment of cord to vibrate, close to two meters. So you may find it useful to put the pole end at one end of one lab bench, and the pulley at the other end of an adjacent lab bench, stretching the cord out diagonally.
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**Measurement plan:** First record the formula you have guessed that relates the speed of sound to the linear density and tension in the cord. Your group should devise a measurement plan to measure get the speed of sound from measurements of frequency and wavelength. Explain how wavelength is found from number of segments at resonance and how this and resonant frequency can give the wave speed. Then set up a plane to compare these measurements with the predictions of your formula. Design a data table. You will want to present the analysis in the form of a graph of predicted versus measured wave speed.

**Implementation:** Implement your plan and record the necessary data.

**Analysis:** Show the analysis of your data, comparing the measured wave speed to the one found from resonance measurements to the formula you predicted for wave speed from linear density and tension. Show details of how both the calculation from resonance data and the calculation from linear density and tension are performed. Prepare your results in the form of a graph.
Conclusions: What have you learned?

1. Summarize in your own words what you have learned in solving these problems—both the problems involving sound and the one involving a string. Get as many of the main points as you can, and describe them as well as you can with one or more sentences each. What aspects of the two types of resonance were the same?

2. Comment on the numerical agreement in the vibrating string study between the wave speed predicted from tension and density and the wave speed actually measured. Explain how you know whether this falls within experimental precision or not.