2023 Math Track Meet

University of North Dakota

February 20, 2023

Contents

INDIVIDUAL TEST #1
Grades 7/8

School __________ Team Name __________

Calculators are allowed. Student Name __________

1. Simplify $(5 \times 8 - 6^2)^3 \div 12$. Your answer should be an integer or a fraction where the numerator and denominator have no common factors.

   (2 pts) 1. ________

2. A class has an ice cream sundae party and every student puts chocolate syrup, strawberry syrup, or both on his or her sundae. Seventeen students take chocolate (with or without strawberry), and 10 take strawberry (with or without chocolate). If 3 students take both chocolate and strawberry, how many students are in the class?

   (3 pts) 2. ________

3. An isosceles triangle has two sides of length 10 cm and one of length 12 cm. What is the area of the triangle in square centimeters?

   (3 pts) 3. ________

4. A fair die has equal probabilities to roll each of the numbers 1 through 6. Two fair dice are thrown. What is the probability that the product of the two numbers rolled is odd?

   (3 pts) 4. ________

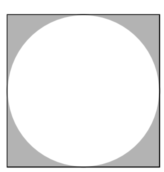
5. A baseball player throws a ball 50 meters in 1.60 seconds. How fast did the ball travel in kilometers per hour? Remember, there are 60 seconds in a minute, 60 minutes in an hour, and 1000 meters in a kilometer.

   (3 pts) 5. ________

6. The 21 students in a morning math class have an average score of 93 on a test. The 18 students in the afternoon class have an average score of 80. What is the average score of all 39 students together?

   (3 pts) 6. ________

7. A circle is inscribed in a square as shown. The square has perimeter 40 cm. What is the area of the circle in square centimeters? Use 3.14 for π.



   (3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #1
Grades 7/8

School ______________

Team Name ______________

Calculators are allowed.

Key

Student Name ______________

1. Simplify $(5 \times 8 - 6^2)^3 \div 12$. Your answer should be an integer or a fraction where the numerator and denominator have no common factors.

(2 pts) 1. $\frac{16}{3}$

2. A class has an ice cream sundae party and every student puts chocolate syrup, strawberry syrup, or both on his or her sundae. Seventeen students take chocolate (with or without strawberry), and 10 take strawberry (with or without chocolate). If 3 students take both chocolate and strawberry, how many students are in the class?

(3 pts) 2. 24

3. An isosceles triangle has two sides of length 10 cm and one of length 12 cm. What is the area of the triangle in square centimeters?

(3 pts) 3. 48

4. A fair die has equal probabilities to roll each of the numbers 1 through 6. Two fair dice are thrown. What is the probability that the product of the two numbers rolled is odd?

(3 pts) 4. $\frac{1}{4}$ or 0.25

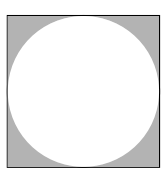
5. A baseball player throws a ball 50 meters in 1.60 seconds. How fast did the ball travel in kilometers per hour? Remember, there are 60 seconds in a minute, 60 minutes in an hour, and 1000 meters in a kilometer.

(3 pts) 5. 112.5

6. The 21 students in a morning math class have an average score of 93 on a test. The 18 students in the afternoon class have an average score of 80. What is the average score of all 39 students together?

(3 pts) 6. 87

7. A circle is inscribed in a square as shown. The square has perimeter 40 cm. What is the area of the circle in square centimeters? Use 3.14 for π.



(3 pts) 7. 78.5

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #2
Grades 7/8

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Student Name ______________

1. Solve for x.

$$|2x - 7| = 3$$

(2 pts) 1. ________

2. Evaluate the expression $-x^2 - 6xy$ given that $x = -2$ and $y = 3$.

(3 pts) 2. ________

3. If Mary builds a race track diagonally across her 6 ft by 8 ft sandbox, how much material, in length, is saved by <u>not</u> going around the two sides ?

(3 pts) 3. ________

4. Jack dug a circular quarry in his sandbox that is 8 inches in diameter on the top and bottom and 5 inches deep. Matt dug a quarry that is 6 inches wide on the top and bottom and 10 inches deep. Which quarry will hold the most water?

(3 pts) 4. ________

5. Evaluate $2\{15 - 3(-4 + 2) + (-1)^5\}$

(3 pts) 5. ________

6. Simplify by adding and subtracting.

$$\frac{3}{7} - \frac{1}{3} - \frac{5}{2}$$

(3 pts) 6. ________

7. Factor completely $16x^4 - 81$

(3 pts) 7. ________

TOTAL POINTS ________

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INDIVIDUAL TEST #2
Grades 7/8

School ______________ Team Name ______________

Calculators are **NOT** allowed. **Key** Student Name ______________

1. Solve for x.

$$|2x - 7| = 3$$

(2 pts) 1. 2,5

2. Evaluate the expression $-x^2 - 6xy$ given that $x = -2$ and $y = 3$.

(3 pts) 2. 32

3. If Mary builds a race track diagonally across her 6 ft by 8 ft sandbox, how much material, in length, is saved by <u>not</u> going around the two sides ?

(3 pts) 3. 4

4. Jack dug a circular quarry in his sandbox that is 8 inches in diameter on the top and bottom and 5 inches deep. Matt dug a quarry that is 6 inches wide on the top and bottom and 10 inches deep. Which quarry will hold the most water?

(3 pts) 4. Matt's

5. Evaluate $2\{15 - 3(-4 + 2) + (-1)^5\}$

(3 pts) 5. 40

6. Simplify by adding and subtracting.

$$\frac{3}{7} - \frac{1}{3} - \frac{5}{2}$$

(3 pts) 6. $-\frac{101}{42}$

7. Factor completely $16x^4 - 81$

(3 pts) 7. $(4x^2+9)(2x-3)(2x+3)$

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #3
Grades 7/8

School ______________ Team Name ______________

Calculators are allowed. Student Name ______________

1. What is the median of the prime factors of 2023^{2023}, including any repeated prime factors.

(2 pts) 1. ________

2. Jack and Jill do a puzzle together in 3 hours. Jill can do the same puzzle on her own in 5 hours. How many hours would it take Jack to do the same puzzle on his own?

(3 pts) 2. ________

3. The area of a square poster is 2023 square inches. What is the side length of the poster, in feet? Round your answer to the nearest hundredth.

(3 pts) 3. ________

4. There are 4 types of cookies on a platter. $\frac{1}{3}$ of the cookies are chocolate chip, $\frac{2}{5}$ of the cookies are monster cookies, 25% of the cookies are sugar cookies, and 3 cookies are oatmeal raisin. How many total cookies are on the platter?

(3 pts) 4. ________

5. At a school gathering, 81 children were asked to get in groups of 3 or 4 people. There were 24 total groups formed. How many groups of 3 children were there?

(3 pts) 5. ________

6. Find the sum of all the odd counting numbers from 1 to 2023.

(3 pts) 6. ________

7. If $\frac{2}{3}$ of a box of chocolate weighs $\frac{3}{4}$ of a pound, then how many pounds does a full box of chocolate weigh? Write your answer as either a whole number, or a simplified mixed number.

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
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INDIVIDUAL TEST #3
Grades 7/8

School ______________

Team Name ______________

Calculators are allowed.

Key

Student Name ______________

1. What is the median of the prime factors of 2023^{2023}, including any repeated prime factors.

(2 pts) 1. 17

2. Jack and Jill do a puzzle together in 3 hours. Jill can do the same puzzle on her own in 5 hours. How many hours would it take Jack to do the same puzzle on his own?

(3 pts) 2. 7.5 or $7\frac{1}{2}$

3. The area of a square poster is 2023 square inches. What is the side length of the poster, in feet? Round your answer to the nearest hundredth.

(3 pts) 3. 3.75

4. There are 4 types of cookies on a platter. $\frac{1}{3}$ of the cookies are chocolate chip, $\frac{2}{5}$ of the cookies are monster cookies, 25% of the cookies are sugar cookies, and 3 cookies are oatmeal raisin. How many total cookies are on the platter?

(3 pts) 4. 180

5. At a school gathering, 81 children were asked to get in groups of 3 or 4 people. There were 24 total groups formed. How many groups of 3 children were there?

(3 pts) 5. 15

6. Find the sum of all the odd counting numbers from 1 to 2023.

(3 pts) 6. $1,024,144$

7. If $\frac{2}{3}$ of a box of chocolate weighs $\frac{3}{4}$ of a pound, then how many pounds does a full box of chocolate weigh? Write your answer as either a whole number, or a simplified mixed number.

(3 pts) 7. $1\frac{1}{8}$

Solutions

1.) What is the median of the prime factors of 2023^{2023}, including any repeated prime factors.

The prime factorization of 2023 is $7 \times 17 \times 17$ or 7×17^2. Thus, the prime factorization of 2023^{2023} is $7^{2023} \times 17^{2023} \times 17^{2023}$ or $7^{2023} \times 17^{4046}$. Consider the list of all the prime factors of 2023^{2023}: 7, 7, … , 7, 7, 17, 17, … 17, 17, which contains 2023 sevens and 4046 seventeens. The median (middle number) of this list of numbers is 17.

Solution: 17

2.) Jack and Jill do a puzzle together in 3 hours. Jill can do the same puzzle on her own in 5 hours. How many hours would it take Jack to do the same puzzle on his own?

Since Jill can do the puzzle in 5 hours on her own, then when working together with Jack, Jill does $\frac{3}{5}$ of the puzzle in 3 hours. This means that Jack does the remaining $\frac{2}{5}$ of the puzzle in 3 hours. At this rate, Jack can do $\frac{1}{5}$ of the puzzle in 1.5 hours, so to complete the whole puzzle ($\frac{5}{5}$ of the puzzle) on his own, he will need $5 \cdot 1.5$ hours, which is 7.5 hours (or $7\frac{1}{2}$ hours).

Solution: 7.5 or $7\frac{1}{2}$

3.) The area of a square poster is 2023 square inches. What is the side length of the poster, in feet? Round your answer to the nearest hundredth.

The formula for the area of a square is $A = s^2$, where A represents the area of a square of side length s. The poster has area 2023 square inches, so substituting this into the area formula, we have $2023 = s^2$. To solve for s, we can take the square root of both sides of the equation, giving us $s = \sqrt{2023} \approx 44.9777$ inches. To convert this to feet, we can divide by 12 since there are 12 inches in 1 foot. This gives us a side length of about 3.75 feet.

Solution: 3.75

4.) There are 4 types of cookies on a platter. $\frac{1}{3}$ of the cookies are chocolate chip, $\frac{2}{5}$ of the cookies are monster cookies, 25% of the cookies are sugar cookies, and 3 cookies are oatmeal raisin. How many total cookies are on the platter?

First, it is worth noting that 25% of the cookies is equivalent to $\frac{1}{4}$ of the cookies. Thus, $\frac{1}{4}$ of the cookies are sugar cookies.

Consider the following table to organize the information.

Cookie type	*Fraction of the total cookies*	*Equivalent fraction of the total cookies*	*Total Number of Cookies*
Chocolate chip	$\frac{1}{3}$	$\frac{1 \cdot 20}{3 \cdot 20} = \frac{20}{60}$	
Monster cookies	$\frac{2}{5}$	$\frac{2 \cdot 12}{5 \cdot 12} = \frac{24}{60}$	
Sugar cookies	$\frac{1}{4}$	$\frac{1 \cdot 15}{4 \cdot 15} = \frac{15}{60}$	
Oatmeal Raisin	x	x	3
All Cookies	1 whole	$\frac{60}{60}$	To be determined!

After finding equivalent fractions with common denominators, we can add the fractions representing each individual type of cookie and their sum should equal the total fraction of cookies: $\frac{20}{60} + \frac{24}{60} + \frac{15}{60} + x = \frac{60}{60}$. Adding the fractions on the left side of the equation gives us $\frac{59}{60} + x = \frac{60}{60}$. Therefore, $x = \frac{1}{60}$. This tells us that $\frac{1}{60}$ of the cookies are oatmeal raisin cookies, and since we know that 3 cookies are oatmeal raisin cookies, then we know that $\frac{1}{60} \cdot (\text{total number of cookies}) = 3$. We can solve this equation by multiplying both sides of the equation by 60, which means that there are 180 cookies total.

Solution: 180

5.) At a school gathering, 81 children were asked to get in groups of 3 or 4 people. There were 24 total groups formed. How many groups of 3 children were there?

Let x represent the number of groups of 3 people. Let y represent the number of groups of 4 people. Then there are $3x$ people in groups of 3 and $4y$ people in groups of 4. Since all 81 children are in groups, then we can add these quantities to get the equation $3x + 4y = 81$. We also know that 24 total groups were formed, which can be though of as the sum of the number of groups of 3 people and the number of groups of 4 people. From this information, we can write the equation $x + y = 24$.

Thus, we are solving the system of equations $\begin{cases} 3x + 4y = 81 \\ x + y = 24 \end{cases}$. Solving the second equation for y gives us $y = 24 - x$. We can now substitute this value of y into the first equation, giving us $3x + 4(24 - x) = 81$. Applying the distributive property, we get $3x + 96 - 4x = 81$. Combining like terms, we get $-x + 96 = 81$. Subtracting 96 from both sides, we get $-x = -15$. Dividing both sides by -1, we get $x = 15$. Recall that x represents the number of groups of 3 people. Therefore, there are 15 groups of 3 children.

Solution: 15

6.) Find the sum of all the odd counting numbers from 1 to 2023.

Rewording the task, we are finding $1 + 3 + 5 + 7 + \cdots + 2017 + 2019 + 2021 + 2023$.

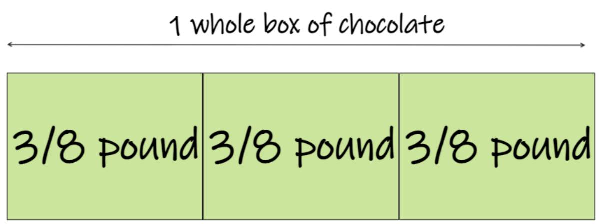
Notice that the first and last number can be added together to get $1 + 2023 = 2024$. The second and second to last numbers can be added together to get $3 + 2021 = 2024$. Continue adding pairs in this manner until you're convinced that all the pairs will add up to 2024. Well, how many pairs do we have? There are 2023 numbers from 1 to 2023, and half of the numbers from 1 to 2022 are odd, and 2023 is odd. That means there are $1011 + 1 = 1012$ odd numbers from 1 to 2023. When we pair the numbers up as previously described, there are $1012 \div 2 = 506$ pairs of numbers. Recall that each pair of numbers has the sum 2024. Therefore, the sum of all the odd counting numbers from 1 to 2023 is $506 \cdot 2024 = 1{,}024{,}144$.

Solution: 1,024,144

7.) If $\frac{2}{3}$ of a box of chocolate weighs $\frac{3}{4}$ of a pound, then how many pounds does a full box of chocolate weigh? Write your answer as either a whole number, or a simplified mixed number.

In brief, the solution to this problem is $\frac{3}{4} pounds \div \frac{2}{3}\ box = \frac{9}{8}\ pounds\ per\ box =$ $1\frac{1}{8}\ pounds\ per\ box$ (because $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \cdot \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$).

To reason through this, we know that $\frac{2}{3}$ of a box weighs $\frac{3}{4}$ of a pound. Equivalently, that is $\frac{6}{8}$ of a pound. Then $\frac{1}{3}$ of a box (which is half of $\frac{2}{3}$ of a box) will weigh $\frac{3}{8}$ of a pound (which is half of $\frac{6}{8}$ of a pound). This means 1 whole box (which is $\frac{3}{3}$ of a box) will weigh $3 \cdot \frac{3}{8} = \frac{9}{8}$ pounds. Consider the following picture to support this explanation.



Solution: $1\frac{1}{8}$

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #4
Grades 7/8

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Student Name ______________

1. Evaluate

$$(-14) \div 2 - 10 \div (-2) - 3|2 - (-7)|$$

(2 pts) 1. ________

2. $\frac{2}{9}$ of the students in a math contest are from seventh grade. If there are 95 more eighth-grade students than seventh-grade students, how many eighth-grade students are there in the math contest?

(3 pts) 2. ________

3. Two standard six-sided dice are thrown, and the dice are considered fair. What is the probability that the sum of two numbers facing up is greater than 10?

(3 pts) 3. ________

4. A cylindrical fish tank is 1 foot tall. The radius of the fish tank is 5 inches. How much water does it take to fill the tank? Do not convert π to a number. Be sure to use the proper label in your answer.

(3 pts) 4. ________

5. A firefighter truck can hold 3000 gallons of water. A firefighter can deliver 160 gallons of water every 2 minutes. How long will it take for the firefighter to empty the tank? Be sure to use the proper label in your answer.

(3 pts) 5. ________

6. Find the x-intercept for

$$f(x) = x^2 - 6x + 9$$

(3 pts) 6. ________

7. Simplify the expression

$$(-10)^0 - (-2)^3 - 3^2 + 2^{-3}$$

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
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INDIVIDUAL TEST #4
Grades 7/8

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Key

Student Name ______________

1. Evaluate
$$(-14) \div 2 - 10 \div (-2) - 3|2 - (-7)|$$

(2 pts) 1. -29

2. $\frac{2}{9}$ of the students in a math contest are from seventh grade. If there are 95 more eighth-grade students than seventh-grade students, how many eighth-grade students are there in the math contest?

(3 pts) 2. 133

3. Two standard six-sided dice are thrown, and the dice are considered fair. What is the probability that the sum of two numbers facing up is greater than 10?

(3 pts) 3. $\frac{1}{12}$

4. A cylindrical fish tank is 1 foot tall. The radius of the fish tank is 5 inches. How much water does it take to fill the tank? Do not convert π to a number. Be sure to use the proper label in your answer.

(3 pts) 4. 300π in^3 or $\frac{25}{144}\pi$ ft^3

5. A firefighter truck can hold 3000 gallons of water. A firefighter can deliver 160 gallons of water every 2 minutes. How long will it take for the firefighter to empty the tank? Be sure to use the proper label in your answer.

(3 pts) 5. 37.5 minutes

6. Find the x-intercept for
$$f(x) = x^2 - 6x + 9$$

(3 pts) 6. 3 or $(3, 0)$

7. Simplify the expression
$$(-10)^0 - (-2)^3 - 3^2 + 2^{-3}$$

(3 pts) 7. $\frac{1}{8}$

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

TEAM TEST #2
Grades 7/8

School ______________

Team Name ______________

Calculators are **NOT** allowed.

1. Gavin and Nancy had a total of 42 markers. After Gavin gave 30% of their markers to Nancy, Nancy had twice as many as Gavin. How many markers did Gavin have originally?

(20 pts) 1. ________

2. There are 37 numbers on a roulette wheel: 0 and the whole numbers 1 to 36. What is the chance of getting a prime number?

(20 pts) 2. ________

3. The average of five weights is 13 grams. If a 7-gram weight is added, what is the average of the six weights?

(20 pts) 3. ________

4. A local shoe company is going out of business. They plan to mark everything down 40% the first week of the sale and then a final markdown of 70% more the last week they are open. How much will a $110 pair of shoes cost after the final markdown?

A) $13.30 B) $19.80 C) $30.80 D) $46.20 E) Not possible.

(20 pts) 4. ________

5. Circle X has a radius of π. Circle Y has a circumference of 8π. Circle Z has an area of 9π. List the circles in order from smallest to largest.

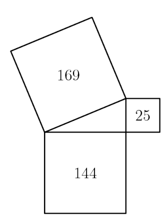
A) X, Y, Z B) Z, Y, X C) Y, Z, X D) Z, X, Y E) Y, X, Z

(20 pts) 5. ________

6. A box contains gold coins. If the coins are equally divided among 6 people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds the smallest number of coins that meet these two conditions, how many coins are left when equally divided among 7 people?

(20 pts) 6. ________

7. Given the areas of the three squares in the figure, what is the area of the interior triangle?



(20 pts) 7. ________

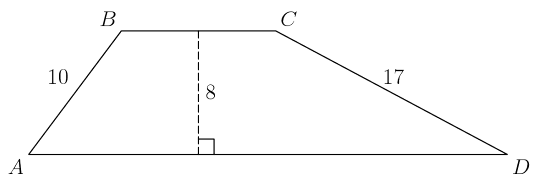
8. How many integers between 1000 and 2023 have all three of the numbers 15, 20 and 25 as factors?

(20 pts) 8. ________

9. Students from Wednesday Addams' class are standing in a circle. They are evenly spaced and consecutively labeled using whole numbers starting from 1. The student in place number 3 is standing directly across from the student in place number 17. How many students are there in Wednesday Addams' class?

(20 pts) 9. ________

10. The area of trapezoid $ABCD$ is 164 cm^2. The altitude is 8 cm, AB is 10 cm, and CD is 17 cm. what is BC, in centimeters?



(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
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TEAM TEST #2
Grades 7/8

School ______________ **Key** Team Name ______________

Calculators are **NOT** allowed.

1. Gavin and Nancy had a total of 42 markers. After Gavin gave 30% of their markers to Nancy, Nancy had twice as many as Gavin. How many markers did Gavin have originally?

(20 pts) 1. 20

2. There are 37 numbers on a roulette wheel: 0 and the whole numbers 1 to 36. What is the chance of getting a prime number?

(20 pts) 2. 11/37 or 11 out of 37

3. The average of five weights is 13 grams. If a 7-gram weight is added, what is the average of the six weights?

(20 pts) 3. 12 grams

4. A local shoe company is going out of business. They plan to mark everything down 40% the first week of the sale and then a final markdown of 70% more the last week they are open. How much will a \$110 pair of shoes cost after the final markdown?

A) \$13.30 B) \$19.80 C) \$30.80 D) \$46.20 E) Not possible.

(20 pts) 4. B

5. Circle X has a radius of π. Circle Y has a circumference of 8π. Circle Z has an area of 9π. List the circles in order from smallest to largest.

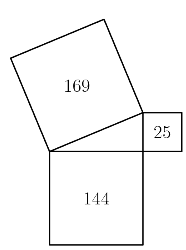
A) X, Y, Z B) Z, Y, X C) Y, Z, X D) Z, X, Y E) Y, X, Z

(20 pts) 5. D

6. A box contains gold coins. If the coins are equally divided among 6 people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds the smallest number of coins that meet these two conditions, how many coins are left when equally divided among 7 people?

(20 pts) 6. 0

7. Given the areas of the three squares in the figure, what is the area of the interior triangle?



(20 pts) 7. 30

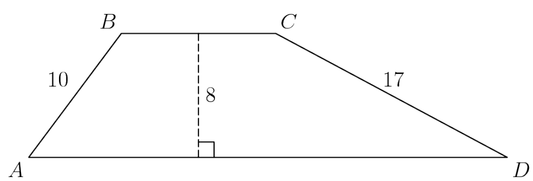
8. How many integers between 1000 and 2023 have all three of the numbers 15, 20 and 25 as factors?

(20 pts) 8. ___3___

9. Students from Wednesday Addams' class are standing in a circle. They are evenly spaced and consecutively labeled using whole numbers starting from 1. The student in place number 3 is standing directly across from the student in place number 17. How many students are there in Wednesday Addams' class?

(20 pts) 9. ___28___

10. The area of trapezoid $ABCD$ is 164 cm^2. The altitude is 8 cm, AB is 10 cm, and CD is 17 cm. what is BC, in centimeters?



(20 pts) 10. ___10___

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School ______________ Team Name ______________

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(20 pts) 1. ________

2. There are 37 numbers on a roulette wheel: 0 and the whole numbers 1 to 36. What is the chance of getting a prime number?

(20 pts) 2. ________

3. The average of five weights is 13 grams. If a 7-gram weight is added, what is the average of the six weights?

(20 pts) 3. ________

4. A local shoe company is going out of business. They plan to mark everything down 40% the first week of the sale and then a final markdown of 70% more the last week they are open. How much will a $110 pair of shoes cost after the final markdown?

A) $13.30 B) $19.80 C) $30.80 D) $46.20 E) Not possible.

(20 pts) 4. ________

5. Circle X has a radius of π. Circle Y has a circumference of 8π. Circle Z has an area of 9π. List the circles in order from smallest to largest.

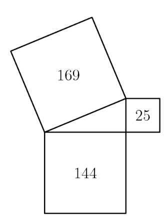
A) X, Y, Z B) Z, Y, X C) Y, Z, X D) Z, X, Y E) Y, X, Z

(20 pts) 5. ________

6. A box contains gold coins. If the coins are equally divided among 6 people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds the smallest number of coins that meet these two conditions, how many coins are left when equally divided among 7 people?

(20 pts) 6. ________

7. Given the areas of the three squares in the figure, what is the area of the interior triangle?



(20 pts) 7. ________

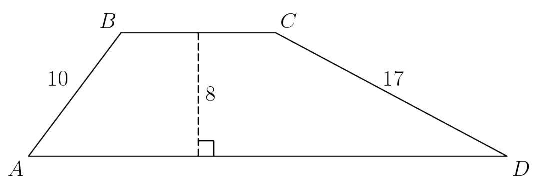
8. How many integers between 1000 and 2023 have all three of the numbers 15, 20 and 25 as factors?

(20 pts) 8. ________

9. Students from Wednesday Addams' class are standing in a circle. They are evenly spaced and consecutively labeled using whole numbers starting from 1. The student in place number 3 is standing directly across from the student in place number 17. How many students are there in Wednesday Addams' class?

(20 pts) 9. ________

10. The area of trapezoid $ABCD$ is 164 cm^2. The altitude is 8 cm, AB is 10 cm, and CD is 17 cm. what is BC, in centimeters?



(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

TEAM TEST #2
Grades 7/8

School ________ **Key** Team Name ________

Calculators are **NOT** allowed.

1. Gavin and Nancy had a total of 42 markers. After Gavin gave 30% of their markers to Nancy, Nancy had twice as many as Gavin. How many markers did Gavin have originally?

(20 pts) 1. 20

2. There are 37 numbers on a roulette wheel: 0 and the whole numbers 1 to 36. What is the chance of getting a prime number?

(20 pts) 2. 11/37 or 11 out of 37

3. The average of five weights is 13 grams. If a 7-gram weight is added, what is the average of the six weights?

(20 pts) 3. 12 grams

4. A local shoe company is going out of business. They plan to mark everything down 40% the first week of the sale and then a final markdown of 70% more the last week they are open. How much will a \$110 pair of shoes cost after the final markdown?

A) \$13.30 B) \$19.80 C) \$30.80 D) \$46.20 E) Not possible.

(20 pts) 4. B

5. Circle X has a radius of π. Circle Y has a circumference of 8π. Circle Z has an area of 9π. List the circles in order from smallest to largest.

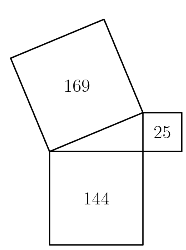
A) X, Y, Z B) Z, Y, X C) Y, Z, X D) Z, X, Y E) Y, X, Z

(20 pts) 5. D

6. A box contains gold coins. If the coins are equally divided among 6 people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds the smallest number of coins that meet these two conditions, how many coins are left when equally divided among 7 people?

(20 pts) 6. 0

7. Given the areas of the three squares in the figure, what is the area of the interior triangle?



(20 pts) 7. 30

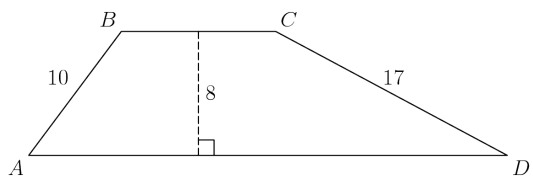
8. How many integers between 1000 and 2023 have all three of the numbers 15, 20 and 25 as factors?

(20 pts) 8. ___3___

9. Students from Wednesday Addams' class are standing in a circle. They are evenly spaced and consecutively labeled using whole numbers starting from 1. The student in place number 3 is standing directly across from the student in place number 17. How many students are there in Wednesday Addams' class?

(20 pts) 9. ___28___

10. The area of trapezoid $ABCD$ is 164 cm^2. The altitude is 8 cm, AB is 10 cm, and CD is 17 cm. what is BC, in centimeters?



(20 pts) 10. ___10___

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #1
Grades 9/10

School ____________

Team Name ____________

Calculators are allowed.

Student Name ____________

1. How many integers between 100 and 300 are multiples of both 5 and 7 but are not multiples of 10?

(2 pts) 1. ________

2. Alice, Betty, and Carol play a two-player game that never ends in a tie. In a recent tournament between the three players, a total of 60 games were played and each of the players played the same number of games. When Alice and Betty played, Alice won 20% of the games. When Betty and Carol played, Betty won 60% of the games. When Carol and Alice played, Carol won 40% of the games. How many games did Betty win?

(3 pts) 2. ________

3. Michelle texted a six-digit integer to Veronica. Two of the digits of the six-digit integer were 6's. Unfortunately, the two 6's that Michelle texted did not appear and Veronica instead received the four-digit integer 2023. What is the number of possible six-digit integers that Michelle could have texted?

(3 pts) 3. ________

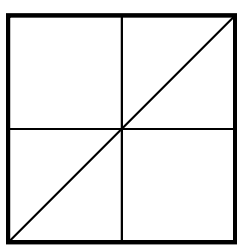
4. The sum of nine consecutive positive integers is 99. What is the largest of these integers?

(3 pts) 4. ________

5. Janet has 10 coins consisting of nickels, dimes, and quarters. Seven of the coins are either dimes or quarters, and eight of the coins are either dimes or nickels. How many dimes does Janet have?

(3 pts) 5. ________

6. In the game "TRISQUARE", three points are awarded for each triangle found, and four points for each square. What is the highest number of points that can be achieved for the given diagram?



(3 pts) 6. ________

7. Each of the numbers 1, 2, 3, and 4 is substituted, in some order, for p, q, r, and s. What is the highest possible value of $p^q + r^s$?

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #1
Grades 9/10

School ______________

Team Name ______________

Calculators are allowed.

Key

Student Name ______________

1. How many integers between 100 and 300 are multiples of both 5 and 7 but are not multiples of 10?

(2 pts) 1. 3

2. Alice, Betty, and Carol play a two-player game that never ends in a tie. In a recent tournament between the three players, a total of 60 games were played and each of the players played the same number of games. When Alice and Betty played, Alice won 20% of the games. When Betty and Carol played, Betty won 60% of the games. When Carol and Alice played, Carol won 40% of the games. How many games did Betty win?

(3 pts) 2. 28

3. Michelle texted a six-digit integer to Veronica. Two of the digits of the six-digit integer were 6's. Unfortunately, the two 6's that Michelle texted did not appear and Veronica instead received the four-digit integer 2023. What is the number of possible six-digit integers that Michelle could have texted?

(3 pts) 3. 15

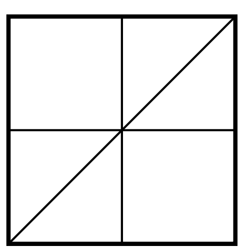
4. The sum of nine consecutive positive integers is 99. What is the largest of these integers?

(3 pts) 4. 15

5. Janet has 10 coins consisting of nickels, dimes, and quarters. Seven of the coins are either dimes or quarters, and eight of the coins are either dimes or nickels. How many dimes does Janet have?

(3 pts) 5. 5

6. In the game "TRISQUARE", three points are awarded for each triangle found, and four points for each square. What is the highest number of points that can be achieved for the given diagram?



(3 pts) 6. 38

7. Each of the numbers 1, 2, 3, and 4 is substituted, in some order, for p, q, r, and s. What is the highest possible value of $p^q + r^s$?

(3 pts) 7. 83

#1 $S(7) = 35$

$\cancel{35}, \cancel{70}, 105, \cancel{140}, 175, \cancel{210}, 245, \cancel{280}, \cancel{315}$

$\boxed{\text{3 values}}$

#2

A & B (20)	B & C (20)	A & C (20)
A = 4	B = 12	A = 12
B = 16	C = 8	C = 8

$\boxed{\text{Betty won 28 games}}$

#3

662023	620263	260623	206623	202663
626023	620236	260263	206263	202636
620623	266023	260236	206236	202366

$\boxed{15}$

#4 $x + x+1 + x+2 + x+3 + x+4 + x+5 + x+6 + x+7 + x+8 = 99$

$9x + 36 = 99$ $\qquad$ $x+8 = 7+8 = \boxed{15}$

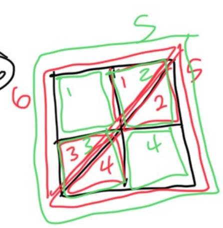
$x = 7$

#5 $N + D + Q = 10$ $\qquad$ $N + 7 = 10$

$D + Q = 7$ $\qquad$ $N = 3$

$D + N = 8$ $\qquad$ $\boxed{D = 5}$

#6



$6(3) = 18$

$5(4) = \underline{20}$

$\boxed{\text{38 points}}$

#7 $3^4 + 2^1 = 81 + 2 = \boxed{83}$

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #2
Grades 9/10

School ____________

Team Name ____________

Calculators are **NOT** allowed.

Student Name ____________

1. What is the product of two roots of the equation $2 + \dfrac{1}{x+3} = x - 1$?

(2 pts) 1. ________

2. $x - 1$ divides $2x^2 + kx + 1$ where k is a constant. What is the sum of all two roots of the equation $2x^2 + kx + 1 = 0$?

(3 pts) 2. ________

3. If $-2 \le x \le 3$ and $-4 \le y \le 5$, then $a \le x^2 y \le b$. What is $a + b$?

(3 pts) 3. ________

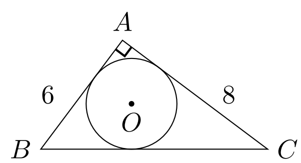
4. If a sequence $\{a_n\}$ of positive numbers converges to L and $a_{n+1}^2 - 6 = a_n$, what is L?

(3 pts) 4. ________

5. If $\big||x-2|+3\big| > 4$, then $x < a$ or $x > b$. What is $a + b$?

(3 pts) 5. ________

6. $\triangle ABC$ ia a right triangle with $\angle A = 90°$, $AB = 6$, and $AC = 8$. What is the radius of the circle with center O inscribed in $\triangle ABC$?



(3 pts) 6. ________

7. A six-sided die is rolled twice. What is the probability that the sum of the outcomes is not 7?
   (a) 4/6 (b) 5/6 (c) 4/7 (d) 5/7 (e) 6/7

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #2
Grades 9/10

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Key

Student Name ______________

1. What is the product of two roots of the equation $2 + \dfrac{1}{x+3} = x - 1$?

(2 pts) 1. -10

2. $x - 1$ divides $2x^2 + kx + 1$ where k is a constant. What is the sum of all two roots of the equation $2x^2 + kx + 1 = 0$?

(3 pts) 2. $\dfrac{3}{2}$

3. If $-2 \le x \le 3$ and $-4 \le y \le 5$, then $a \le x^2 y \le b$. What is $a + b$?

(3 pts) 3. 9

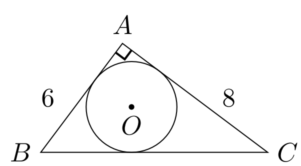
4. If a sequence $\{a_n\}$ of positive numbers converges to L and $a_{n+1}^2 - 6 = a_n$, what is L?

(3 pts) 4. 3

5. If $\big||x-2| + 3\big| > 4$, then $x < a$ or $x > b$. What is $a + b$?

(3 pts) 5. 4

6. $\triangle ABC$ ia a right triangle with $\angle A = 90°$, $AB = 6$, and $AC = 8$. What is the radius of the circle with center O inscribed in $\triangle ABC$?



(3 pts) 6. 2

7. A six-sided die is rolled twice. What is the probability that the sum of the outcomes is not 7?
   (a) 4/6 (b) 5/6 (c) 4/7 (d) 5/7 (e) 6/7

(3 pts) 7. (b)

Individual Test #2 w/o calculator (dh)

1. What is the product of two roots of the equation $2 + \dfrac{1}{x+3} = x - 1$?
   (sol)
   $(x+3)(x-3) = 1 \implies x^2 - 10 = 0.$ 1. $\boxed{-10}$

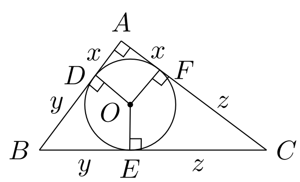
2. $x - 1$ divides $2x^2 + kx + 1$ where k is a constant. What is the sum of two roots of the equation $2x^2 + kx + 1 = 0$?
   (sol)
   $2x^2 + kx + 1 = (x-1)(2x+k+2) + k + 3 \implies k = -3 \implies x^2 - \frac{3}{2} + \frac{1}{2} = 0.$ 2. $\boxed{\dfrac{3}{2}}$

3. If $-2 \le x \le 3$ and $-4 \le y \le 5$, then $a \le x^2 y \le b$. What is $a + b$?
   (sol)
   $-2 \le x \le 3 \implies 0 \le x^2 \le 9 \implies -36 \le x^2 y \le 45.$ 3. $\boxed{9}$

4. If a sequence $\{a_n\}$ of positive numbers converges to L and $a_{n+1}^2 - 6 = a_n$, what is L?
   (sol)
   $L^2 - 6 = L \implies (L-3)(L+2) = 0$. Existence of such a sequence can be shown. 4. $\boxed{L = 3}$

5. If $\big||x-2|+3\big| > 4$, then $x < a$ or $x > b$. What is $a + b$?
   (sol)
   $\big||x-2|+3\big| > 4 \implies \underbrace{|x-2|+3 < -4}_{\text{not possible}}$ or $|x-2|+3 > 4 \implies |x-2| > 1$
   $\implies x - 2 < -1$ or $x - 2 > 1 \implies x < 1$ or $x > 3.$ 5. $\boxed{4}$

6. $\triangle ABC$ ia a right triangle with $\angle A = 90°$, $AB = 6$, and $AC = 8$. What is the radius of the circle with center O inscribed in $\triangle ABC$?



   (sol) $x + y = 6$, $x + z = 8$, $y + z = 10$ by Pythagorean identity
   $\implies x = 2$, $y = 4$, $z = 6$. 6. $\boxed{2}$

7. A six-sided die is rolled twice. What is the probability that the sum of the outcomes is not 7?
   (a) 4/6 (b) 5/6 (c) 4/7 (d) 5/7 (e) 6/7
   (sol)
   Total # of outcomes 36, # of outcomes of the sum being 7 is 6, probability of getting sum 7 is 6/36=1/6. 7. $\boxed{\text{(b)}}$

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #3
Grades 9/10

School ______________

Team Name ______________

Calculators are allowed.
All answers should be exact or rounded to three significant figures.

Student Name ______________

1. How many four digit PINs can be created if no digit is repeated? (2pts) 1. ________

2. What is the probability that if you flip a coin six times, there are an even number of tails? (3pts) 2. ________

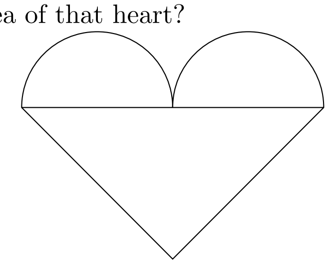
3. Find the angle in degrees between the minute and hour hands of a clock when the time is 5:00. (3pts) 3. ________

4. Alice and Beth can shovel a driveway in one hour. Working alone, it would take Alice 90 minutes. How long would it take Beth working alone? (3pts) 4. ________

5. A house has people with two legs and two eyes; and dogs with four legs and two eyes. Combined, there are 14 legs and 10 eyes. How many dogs are there? (3pts) 5. ________

6. What is the sum of the distinct prime factors of the number 2023? (3pts) 6. ________

7. We can make a heart by placing two semicircles of radius 1 atop a right triangle as below. What is the area of that heart?



(3pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #3
Grades 9/10

SOLUTION KEY **SOLUTION KEY** **SOLUTION KEY**

School ____________

Team Name ____________

Calculators are allowed.
All answers should be exact or rounded to three significant figures.

Student Name ____________

1. How many four digit PINs can be created if no digit is repeated? (2pts) 1. ________ $10 \cdot 9 \cdot 8 \cdot 7 = 5040$

2. What is the probability that if you flip a coin six times, there are an even number of tails? (3pts) 2. ________ $\frac{32}{64} = \frac{1}{2} = 0.5$
   Hint: $(\binom{6}{0} + \binom{6}{2} + \binom{6}{4} + \binom{6}{6})/2^6$ *(also, when you get to the last coin, there are either an even or an odd number of tails so far)*

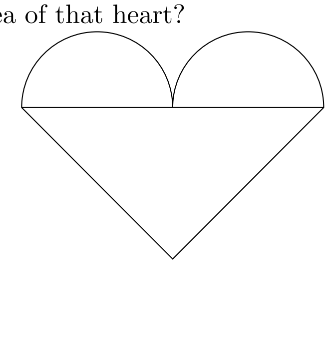
3. Find the angle in degrees between the minute and hour hands of a clock when the time is 5:00. (3pts) 3. ________ $150°$

4. Alice and Beth can shovel a driveway in one hour. Working alone, it would take Alice 90 minutes. How long would it take Beth working alone? (3pts) 4. ________ 3 hours or 180 mins
   Hint: Alice can shovel 2/3 of the driveway in one hour

5. A house has people with two legs and two eyes; and dogs with four legs and two eyes. Combined, there are 14 legs and 10 eyes. How many dogs are there? (3pts) 5. ________ 2
   Hint: 3 people and 2 dogs

6. What is the sum of the distinct prime factors of the number 2023? (3pts) 6. ________ 24
   Hint: $7 + 17$

7. We can make a heart by placing two semicircles of radius 1 atop a right triangle as below. What is the area of that heart?



(3pts) 7. ________ $4 + \pi \approx 7.14$

SOLUTION KEY **SOLUTION KEY** **SOLUTION KEY**

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #4
Grades 9/10

School ______________

Team Name ______________

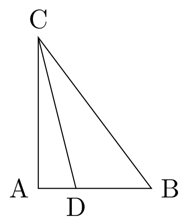
Calculators are **NOT** allowed.

Student Name ______________

1. Simplify $\frac{1}{x}\left(x+\frac{1}{x}\right)^{-1}$

(2 pts) 1. ________

2. In the figure, $\angle CAB$ is a right angle, D lies in the line segment $\overline{AB}$, the length of $\overline{AC}$ is 8, the length of $\overline{BC}$ is 10, and the length of $\overline{BD}$ is 4. What is the length of $\overline{CD}$?



(3 pts) 2. ________

3. In a round-robin tennis tournament, each player plays every other player exactly one time. The number of matches M is given by $M = \frac{1}{2}n(n-1)$ where n is the number of players in the tournament. If there are 45 matches played, how many players were in the tournament?

(3 pts) 3. ________

4. Solve for x. $|x + |3x - 2|| = 2$

(3 pts) 4. ________

5. A room is 30 feet square and 12 feet high. A spider is located in one of the corners on the floor. An unsuspecting fly rests at the diagonally opposite corner on the ceiling. If the fly does not move, what is the shortest distance the spider must crawl along the wals to catch the fly?

(3 pts) 5. ________

6. A ball is thrown from an initial height of 49 feet with an initial velocity of 3 ft/s. The height of the ball in feet after t seconds is given by $h(t) = 49 + 3t - t^2$. What is the maximum height the ball will reach?

(3 pts) 6. ________

7. Solve for x. $\log_4 x = 3 - \log_4(x - 12)$

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #4
Grades 9/10

School ____________

Team Name ____________

Calculators are **NOT** allowed.

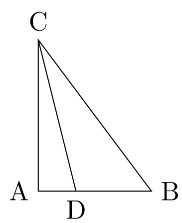
Key

Student Name ____________

1. Simplify $\frac{1}{x}\left(x+\frac{1}{x}\right)^{-1}$

(2 pts) 1. $\frac{1}{x^2+1}$

2. In the figure, $\angle CAB$ is a right angle, D lies in the line segment $\overline{AB}$, the length of $\overline{AC}$ is 8, the length of $\overline{BC}$ is 10, and the length of $\overline{BD}$ is 4. What is the length of $\overline{CD}$?



(3 pts) 2. $2\sqrt{17}$

3. In a round-robin tennis tournament, each player plays every other player exactly one time. The number of matches M is given by $M = \frac{1}{2}n(n-1)$ where n is the number of players in the tournament. If there are 45 matches played, how many players were in the tournament?

(3 pts) 3. 10

4. Solve for x. $|x + |3x - 2|| = 2$

(3 pts) 4. $0, 1$

5. A room is 30 feet square and 12 feet high. A spider is located in one of the corners on the floor. An unsuspecting fly rests at the diagonally opposite corner on the ceiling. If the fly does not move, what is the shortest distance (in feet) the spider must crawl along the walls to catch the fly?

(3 pts) 5. $30\sqrt{2}+12$

6. A ball is thrown from an initial height of 49 feet with an initial velocity of 3 ft/s. The height of the ball in feet after t seconds is given by $h(t) = 49 + 3t - t^2$. What is the maximum height the ball will reach?

(3 pts) 6. 51.25 ft or $\frac{205}{4}$ ft

7. Solve for x. $\log_4 x = 3 - \log_4(x - 12)$

(3 pts) 7. 16

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

TEAM TEST # 2
Grades 9/10

School ______________ Team Name ______________

Calculators are **NOT** allowed.

1. Find the area of a sector of a circle with radius 8 inches subtended by a central angle of $\dfrac{\pi}{2}$ radians.

(20 pts) 1. ________

2. Solve the equation $12 - 2\log_{10}(8x+4) = 8$.

(20 pts) 2. ________

3. The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5050 is collected. How many more children than adults attended?

(20 pts) 3. ________

4. If the cost of a bat and a baseball combined is \$1.10 and the bat costs \$1.00 more than the ball, how much is the ball?

(20 pts) 4. ________

5. Let ℓ_1 be the line segment whose endpoints lie at the top left and bottom right corner of a square and let ℓ_2 be the line segment whose endpoints are the bottom left corner of the square and the center of the top side of the square. What fraction of the area of the square is occupied by the triangle formed from the point at the intersection of ℓ_1 and ℓ_2 and the bottom two corners of the square?

(20 pts) 5. ________

6. The square root of the value obtained by adding two to some positive number is the same as the number itself. What is it?

(20 pts) 6. ________

7. A movie theater in a small town usually opens its doors 3 days in a row and then closes the next day for maintenance. Another movie theater is open 4 days in a row and then closes the next day for the same reason. Suppose both movie theaters are closed today and that today is Wednesday. What day is it the next time they are both closed again at the same time?

(20 pts) 7. ________

8. The sum of the squares of two consecutive positive odd integers is 74. What is the value of the smaller integer?

(20 pts) 8. ________

9. A parallelogram with sides of length x and $\frac{3x+2}{2}$ has perimeter 32. What is the length of the longer side?

(20 pts) 9. ________

10. How many ways are there to pick two nonconsecutive numbers from the first 50 positive integers?

(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET | TEAM TEST # 2
University of North Dakota | Grades 9/10
February 20, 2023

School ____________ **Key** Team Name ____________

Calculators are **NOT** allowed.

1. Find the area of a sector of a circle with radius 8 inches subtended by a central angle of $\frac{\pi}{2}$ radians.

(20 pts) 1. 16π in^2

2. Solve the equation $12 - 2\log_{10}(8x + 4) = 8$.

(20 pts) 2. 12

3. The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5050 is collected. How many more children than adults attended?

(20 pts) 3. 800

4. If the cost of a bat and a baseball combined is \$1.10 and the bat costs \$1.00 more than the ball, how much is the ball?

(20 pts) 4. \$0.05

5. Let ℓ_1 be the line segment whose endpoints lie at the top left and bottom right corner of a square and let ℓ_2 be the line segment whose endpoints are the bottom left corner of the square and the center of the top side of the square. What fraction of the area of the square is occupied by the triangle formed from the point at the intersection of ℓ_1 and ℓ_2 and the bottom two corners of the square?

(20 pts) 5. $\frac{1}{3}$

6. The square root of the value obtained by adding two to some positive number is the same as the number itself. What is it?

(20 pts) 6. 2

7. A movie theater in a small town usually opens its doors 3 days in a row and then closes the next day for maintenance. Another movie theater is open 4 days in a row and then closes the next day for the same reason. Suppose both movie theaters are closed today and that today is Wednesday. What day is it the next time they are both closed again at the same time?

(20 pts) 7. Tuesday

8. The sum of the squares of two consecutive positive odd integers is 74. What is the value of the smaller integer?

(20 pts) 8. 5

9. A parallelogram with sides of length x and $\frac{3x+2}{2}$ has perimeter 32. What is the length of the longer side?

(20 pts) 9. 10

10. How many ways are there to pick two nonconsecutive numbers from the first 50 positive integers?

(20 pts) 10. 1176

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

TEAM TEST # 2
Grades 9/10

Answer Key

1. Find the area of a sector of a circle with radius 8 inches subtended by a central angle of $\frac{\pi}{2}$ radians.

   The area of a sector of a circle with radius r subtended by an angle θ, measured in radians, is $A = \frac{1}{2}\theta r^2$. So,

$$A = \frac{1}{2}\left(\frac{\pi}{2}\right)(8 \text{ inches})^2 = 16\pi \text{ inches}^2.$$

   Alternatively, one could simply note that this sector is a fourth of the circle and divide the area of the entire circle by 4 using the well-known formula $A = \pi r^2$ for the area of a circle.

2. Solve the equation $12 - 2\log_{10}(8x+4) = 8$.

$$\begin{aligned} 12 - 2\log_{10}(8x+4) &= 8 \\ -2\log_{10}(8x+4) &= -4 \\ \log_{10}(8x+4) &= 2 \\ 10^{\log_{10}(8x+4)} &= 10^2 \\ 8x+4 &= 100 \\ 8x &= 96 \\ x &= 12 \end{aligned}$$

3. The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5050 is collected. How many more children than adults attended?

   Let x denote the number of adults and let y denote the number of children. Then, we have the system

$$\begin{cases} x + y &= 2200 \\ 4x + 1.5y &= 5050 \end{cases}$$

   So, we solve the system for x and y using whatever method seems easiest. Here, we opt for substitution and solve the first equation for x:

$$\begin{aligned} x + y &= 2200 \\ x &= 2200 - y \end{aligned}$$

   Now, we substitute our formula for x in to the second equation

$$\begin{aligned} 4x + 1.5y &= 5050 \\ 4(2200 - y) + 1.5y &= 5050 \\ 8800 - 2.5y &= 5050 \\ y &= \frac{5050 - 8800}{-2.5} = 1500. \end{aligned}$$

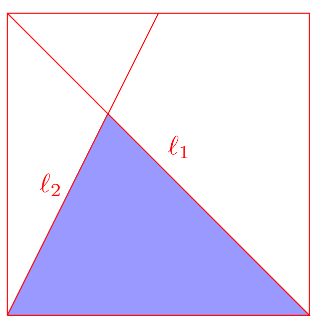
Using our equation from the first step, this gives $x = 2200 - 1500 = 700$. So, we have our answer: 700 adults and 1500 children attended the fair, and $1500 - 700 = 800$ more children attended.

4. If the cost of a bat and a baseball combined is \$1.10 and the bat costs \$1.00 more than the ball, how much is the ball?
   Let x denote the cost of the bat and y denote the cost of the ball. We know that $x + y = \$1.10$ and $x = y + \$1.00$. Combining the second equation with the first, we produce

$$\begin{aligned} x + y &= \$1.10 \\ (y + \$1.00) + y &= \$1.10 \\ 2y + \$1.00 &= \$1.10 \\ 2y &= \$0.10 \\ y &= \$0.05. \end{aligned}$$

   Therefore, the ball costs \$0.05.

5. Let ℓ_1 be the line segment whose endpoints lie at the top left and bottom right corner of a square and let ℓ_2 be the line segment whose endpoints are the bottom left corner of the square and the center of the top side of the square. What fraction of the area of the square is occupied by the triangle formed from the point at the intersection of ℓ_1 and ℓ_2 and the bottom two corners of the square?
   Consider a square of side length 1 for simplicity, so that the area of the triangle is the same as the proportion of the square's area that it occupies. The triangle formed is shaded in blue below:



   Notice that the height of the triangle and the triangle formed above it sum to 1, and that these two triangles are similar. Let h be the height of the larger triangle, so that $1 - h$ is the height of the smaller. As the ratio of the height to the base of similar triangles is the same, we then have

$$\begin{aligned} \frac{h}{1} &= \frac{1-h}{1/2} \\ h &= 2 - 2h \\ h &= \frac{2}{3}. \end{aligned}$$

   So, the area of the triangle is $\frac{1}{2}(1)(h) = \frac{1}{3}$, which is also the proportion of the area of any square a triangle formed in such a fashion will occupy.

6. The square root of the value obtained by adding two to some positive number is the same as the number itself. What is it?
Let x denote our sought value. Then, $\sqrt{x+2} = x$, and we have

$$\begin{aligned} \sqrt{x+2} &= x \\ x+2 &= x^2 \\ x^2 - x - 2 &= 0 \\ x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)} \\ x &= \frac{1 \pm \sqrt{1+8}}{2} \\ x &= \frac{1 \pm 3}{2} \\ x &= 2 \text{ or } -1 \end{aligned}$$

As the number is specified to be positive, we may conclude that it is 2.

7. A movie theater in a small town usually opens its doors 3 days in a row and then closes the next day for maintenance. Another movie theater is open 4 days in a row and then closes the next day for the same reason. Suppose both movie theaters are closed today and that today is Wednesday. What day is it the next time they are both closed again at the same time?
The first theater has an open-closed cycle that repeats every 4 days, while the second has an open-closed cycle that repeats every 5 days. The least common multiple of 4 and 5 is 20, so we need only note that 20 days after Wednesday is Tuesday. That is, the next time both theaters will be closed at once will occur on a Tuesday.

8. The sum of the squares of two consecutive positive odd integers is 74. What is the value of the smaller integer?
Every odd integer is of the form $2k+1$ for some integer $k \geq 0$. So, the sum of the squares of any two consecutive odd integers is of the form $(2k+1)^2 + (2k+3)^2 = 8k^2 + 16k + 10$. If the value of this sum is to be 74, then we need only solve $8k^2 + 16k + 10 = 74$, which amounts to finding the roots of the quadratic $8k^2 + 16k - 64 = 0$, or (factoring out an 8) $k^2 + 2k - 8 = 0$. Via the quadratic formula, this produces $k = -4$ or $k = 2$. Since we are looking for a positive odd integer, we may conclude that $k = 2$. Thus, the smaller integer is $2(2) + 1 = 5$.

9. A parallelogram with sides of length x and $\frac{3x+2}{2}$ has perimeter 32. What is the length of the longer side?
The perimeter of a parallelogram with sides of length x and $\frac{3x+2}{2}$ is $2(x) + 2\left(\frac{3x+2}{2}\right) = 5x + 2$. Therefore, we may conclude that $5x + 2 = 32$ and solve for x to produce $x = 6$. Correspondingly, the sides of the parallelogram are of length $x = 6$ and $\frac{3x+2}{2} = \frac{3(6)+2}{2} = 10$. Thus, the longer side is of length 10.

10. How many ways are there to pick two nonconsecutive numbers from the first 50 positive integers?
There are $\binom{50}{2} = \frac{50!}{2!(50-2)!} = \frac{(50)(49)(48!)}{(2)(48!)} = (25)(49) = 1225$ ways to choose two numbers from the first 50 positive integers. As a set of two consecutive integers $\{x, x+1\}$ is determined uniquely by the smaller of the two values (i.e. by x), notice that there are $50 - 1 = 49$ possible choices for this smallest value (as $50 + 1 = 51$ is not in the first 50 positive integers). So, there are 49 ways to choose two consecutive numbers from the first 50 positive integers, and any of the other 1225 possible ways to choose two numbers from the first 50 positive integers other than these 49 ways results in a pair of nonconsecutive values. Thus, we reach our answer: There are $1225 - 49 = 1176$ ways to pick two nonconsecutive numbers from the first 50 positive integers.

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

TEAM TEST # 2
Grades 9/10

School ______________ Team Name ______________

Calculators are **NOT** allowed.

1. Find the area of a sector of a circle with radius 8 inches subtended by a central angle of $\frac{\pi}{2}$ radians.

(20 pts) 1. ________

2. Solve the equation $12 - 2\log_{10}(8x + 4) = 8$.

(20 pts) 2. ________

3. The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5050 is collected. How many more children than adults attended?

(20 pts) 3. ________

4. If the cost of a bat and a baseball combined is \$1.10 and the bat costs \$1.00 more than the ball, how much is the ball?

(20 pts) 4. ________

5. Let ℓ_1 be the line segment whose endpoints lie at the top left and bottom right corner of a square and let ℓ_2 be the line segment whose endpoints are the bottom left corner of the square and the center of the top side of the square. What fraction of the area of the square is occupied by the triangle formed from the point at the intersection of ℓ_1 and ℓ_2 and the bottom two corners of the square?

(20 pts) 5. ________

6. The square root of the value obtained by adding two to some positive number is the same as the number itself. What is it?

(20 pts) 6. ________

7. A movie theater in a small town usually opens its doors 3 days in a row and then closes the next day for maintenance. Another movie theater is open 4 days in a row and then closes the next day for the same reason. Suppose both movie theaters are closed today and that today is Wednesday. What day is it the next time they are both closed again at the same time?

(20 pts) 7. ________

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(20 pts) 8. ________

9. A parallelogram with sides of length x and $\frac{3x+2}{2}$ has perimeter 32. What is the length of the longer side?

(20 pts) 9. ________

10. How many ways are there to pick two nonconsecutive numbers from the first 50 positive integers?

(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET | TEAM TEST # 2
University of North Dakota | Grades 9/10
February 20, 2023

School ______________ **Key** Team Name ______________

Calculators are **NOT** allowed.

1. Find the area of a sector of a circle with radius 8 inches subtended by a central angle of $\frac{\pi}{2}$ radians.

(20 pts) 1. 16π in^2

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4. If the cost of a bat and a baseball combined is \$1.10 and the bat costs \$1.00 more than the ball, how much is the ball?

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(20 pts) 5. $\frac{1}{3}$

6. The square root of the value obtained by adding two to some positive number is the same as the number itself. What is it?

(20 pts) 6. 2

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(20 pts) 10. 1176

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

TEAM TEST # 2
Grades 9/10

Answer Key

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3. The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5050 is collected. How many more children than adults attended?
   Let x denote the number of adults and let y denote the number of children. Then, we have the system
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Using our equation from the first step, this gives $x = 2200 - 1500 = 700$. So, we have our answer: 700 adults and 1500 children attended the fair, and $1500 - 700 = 800$ more children attended.

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$$\begin{aligned} x + y &= \$1.10 \\ (y + \$1.00) + y &= \$1.10 \\ 2y + \$1.00 &= \$1.10 \\ 2y &= \$0.10 \\ y &= \$0.05. \end{aligned}$$

   Therefore, the ball costs \$0.05.

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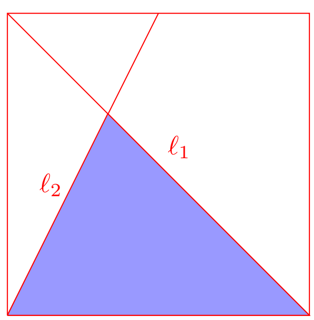

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$$\begin{aligned} \frac{h}{1} &= \frac{1-h}{1/2} \\ h &= 2 - 2h \\ h &= \frac{2}{3}. \end{aligned}$$

   So, the area of the triangle is $\frac{1}{2}(1)(h) = \frac{1}{3}$, which is also the proportion of the area of any square a triangle formed in such a fashion will occupy.

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Let x denote our sought value. Then, $\sqrt{x+2} = x$, and we have

$$\begin{aligned}
\sqrt{x+2} &= x \\
x+2 &= x^2 \\
x^2 - x - 2 &= 0 \\
x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)} \\
x &= \frac{1 \pm \sqrt{1+8}}{2} \\
x &= \frac{1 \pm 3}{2} \\
x &= 2 \text{ or } -1
\end{aligned}$$

As the number is specified to be positive, we may conclude that it is 2.

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9. A parallelogram with sides of length x and $\frac{3x+2}{2}$ has perimeter 32. What is the length of the longer side?
The perimeter of a parallelogram with sides of length x and $\frac{3x+2}{2}$ is $2(x) + 2\left(\frac{3x+2}{2}\right) = 5x + 2$. Therefore, we may conclude that $5x + 2 = 32$ and solve for x to produce $x = 6$. Correspondingly, the sides of the parallelogram are of length $x = 6$ and $\frac{3x+2}{2} = \frac{3(6)+2}{2} = 10$. Thus, the longer side is of length 10.

10. How many ways are there to pick two nonconsecutive numbers from the first 50 positive integers?
There are $\binom{50}{2} = \frac{50!}{2!(50-2)!} = \frac{(50)(49)(48!)}{(2)(48!)} = (25)(49) = 1225$ ways to choose two numbers from the first 50 positive integers. As a set of two consecutive integers $\{x, x+1\}$ is determined uniquely by the smaller of the two values (i.e. by x), notice that there are $50-1=49$ possible choices for this smallest value (as $50+1=51$ is not in the first 50 positive integers). So, there are 49 ways to choose two consecutive numbers from the first 50 positive integers, and any of the other 1225 possible ways to choose two numbers from the first 50 positive integers other than these 49 ways results in a pair of nonconsecutive values. Thus, we reach our answer: There are $1225 - 49 = 1176$ ways to pick two nonconsecutive numbers from the first 50 positive integers.

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #1
Grades 11/12

School ______________

Team Name ______________

Calculators are allowed.

Student Name ______________

1. Find the value of x satisfying $\log_2(3x+2) = 2 + \log_2(x-2)$.

(2 pts) 1. ________

2. Suppose $x^n + x^{-n} = 2023$. What is the exact value of $x^{2n} + x^{-2n}$?

(3 pts) 2. ________

3. A bag has 5 white marbles and 9 black marbles. Three marbles are drawn at random from the bag and not replaced. What is the probability that at least one white marble is picked? Express your answer as a simple fraction in lowest terms.

(3 pts) 3. ________

4. Let a, b, and c be the roots of the polynomial $1000x^3 - 100x^2 + 10x - 1$. What is the value of $(a+1)(b+1)(c+1)$?

(3 pts) 4. ________

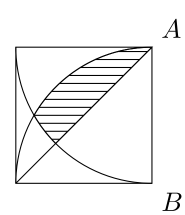
5. Find the minimum possible value of the expression $x^2 + 4x + 2y^2 - 14y + 1$ if x and y are real.

(3 pts) 5. ________

6. John has exactly \$18.90 in dimes and quarters, with twice as many as dimes as quarters. He spends five quarters and twice as many dimes at the convenience store, and he spends 55 cents at the donut shop. If he pays the exact amount for everything, how many quarters does John have left?

(3 pts) 6. ________

7. In the diagram, the curved paths are arcs of circles centered at vertices A and B of a square of side 6. Find the area of the shaded section. Write an exact answer in terms of π or round your answer to two decimal places.



(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #1
Grades 11/12

School ________________

Team Name ________________

Calculators are allowed.

Key

Student Name ________________

1. Find the value of x satisfying $\log_2(3x+2) = 2 + \log_2(x-2)$.

(2 pts) 1. 10

2. Suppose $x^n + x^{-n} = 2023$. What is the exact value of $x^{2n} + x^{-2n}$?

(3 pts) 2. 4092527

3. A bag has 5 white marbles and 9 black marbles. Three marbles are drawn at random from the bag and not replaced. What is the probability that at least one white marble is picked? Express your answer as a simple fraction in lowest terms.

(3 pts) 3. $\frac{10}{13}$

4. Let a, b, and c be the roots of the polynomial $1000x^3 - 100x^2 + 10x - 1$. What is the value of $(a+1)(b+1)(c+1)$?

(3 pts) 4. 1.111

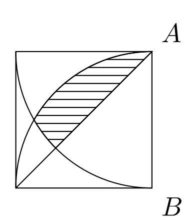
5. Find the minimum possible value of the expression $x^2 + 4x + 2y^2 - 14y + 1$ if x and y are real.

(3 pts) 5. $-\frac{55}{2}$ or -27.5

6. John has exactly \$18.90 in dimes and quarters, with twice as many as dimes as quarters. He spends five quarters and twice as many dimes at the convenience store, and he spends 55 cents at the donut shop. If he pays the exact amount for everything, how many quarters does John have left?

(3 pts) 6. 36

7. In the diagram, the curved paths are arcs of circles centered at vertices A and B of a square of side 6. Find the area of the shaded section. Write an exact answer in terms of π or round your answer to two decimal places.



(3 pts) 7. $\frac{15\pi}{2} - 9\sqrt{3}$ or 7.97

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #2
Grades 11/12

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Student Name ______________

1. Let x be a real number. If $3^{x^4} = 81$, then x^2 is

   (a) -2 (b) 2 (c) 3 (d) 4 (e) not uniquely determined

   (2 pts) 1. ________

2. The 25 cells of square of a 5 by 5 grid are filled with 1's, 2's and 3's, with each cell getting exactly one number, and so that no two cells touching each other either horizontally or vertically get the same number. The numbers in the outside cells are shown. What is the number in the center cell?

1	2	3	1	2
2				3
1		?		2
3				1
2	3	1	2	3

   (a) 1 (b) 2 (c) 3 (d) no number is possible (e) more than one is possible

   (3 pts) 2. ________

3. For how many integers n is the fraction $\frac{6300}{n}$ the square of an integer?

   (a) 4 (b) 7 (c) 8 (d) 14 (e) 16

   (3 pts) 3. ________

4. Let $g(k) = 2k^2$ for any positive integer k. Let $p \geq 3$ be a prime number. The number of positive integers dividing evenly into $g(g(p))$ is

   (a) 7 (b) 8 (c) 15 (d) 20 (e) depends on p

   (3 pts) 4. ________

5. At a daycare with 100 children, each child can bring at most 10 toys (but could also bring fewer than 10 toys, including no toys at all). Without knowing how many toys each child will bring, what is the maximum value of n for which the following statement is true?

   At least n children must bring the same number of toys.

   (a) 9 (b) 10 (c) 11 (d) 12 (e) 100

   (3 pts) 5. ________

6. Let m, n be positive integers such that $\frac{1}{m} + \frac{1}{n} = \frac{19}{94}$. The value of $m + n$ is equal to

   (a) 19 (b) 180 (c) 345 (d) 475 (e) 570

   (3 pts) 6. ________

7. In a triangle $\triangle ABC$, we have the segment lengths $\ell(\overline{AB}) = 4$, $\ell(\overline{AC}) = 2\sqrt{23}$, the measure of the angle $\angle BAC = 46°$. D is a point on segment $\overline{AC}$ such that $\ell(\overline{AD}) = \sqrt{23} + 2$. And, M is the midpoint of $\overline{BC}$. The measure of angle $\angle ADM$ is:

   (a) $65°$ (b) $66°$ (c) $67°$ (d) $68°$ (e) $69°$

   (3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #2
Grades 11/12

School ______________ Team Name ______________

Calculators are **NOT** allowed. **Key** Student Name ______________

1. Let x be a real number. If $3^{x^4} = 81$, then x^2 is

(a) -2 (b) 2 (c) 3 (d) 4 (e) not uniquely determined

(2 pts) 1. B

2. The 25 cells of square of a 5 by 5 grid are filled with 1's, 2's and 3's, with each cell getting exactly one number, and so that no two cells touching each other either horizontally or vertically get the same number. The numbers in the outside cells are shown. What is the number in the center cell?

1	2	3	1	2
2				3
1		?		2
3				1
2	3	1	2	3

(a) 1 (b) 2 (c) 3 (d) no number is possible (e) more than one is possible

(3 pts) 2. C

3. For how many integers n is the fraction $\frac{6300}{n}$ the square of an integer?

(a) 4 (b) 7 (c) 8 (d) 14 (e) 16

(3 pts) 3. C

4. Let $g(k) = 2k^2$ for any positive integer k. Let $p \geq 3$ be a prime number. The number of positive integers dividing evenly into $g(g(p))$ is

(a) 7 (b) 8 (c) 15 (d) 20 (e) depends on p

(3 pts) 4. D

5. At a daycare with 100 children, each child can bring at most 10 toys (but could also bring fewer than 10 toys, including no toys at all). Without knowing how many toys each child will bring, what is the maximum value of n for which the following statement is true?

At least n children must bring the same number of toys.

(a) 9 (b) 10 (c) 11 (d) 12 (e) 100

(3 pts) 5. B

6. Let m, n be positive integers such that $\frac{1}{m} + \frac{1}{n} = \frac{19}{94}$. The value of $m + n$ is equal to

(a) 19 (b) 180 (c) 345 (d) 475 (e) 570

(3 pts) 6. D

7. In a triangle $\triangle ABC$, we have the segment lengths $\ell(\overline{AB}) = 4$, $\ell(\overline{AC}) = 2\sqrt{23}$, the measure of the angle $\angle BAC = 46°$. D is a point on segment $\overline{AC}$ such that $\ell(\overline{AD}) = \sqrt{23} + 2$. And, M is the midpoint of $\overline{BC}$. The measure of angle $\angle ADM$ is:

(a) $65°$ (b) $66°$ (c) $67°$ (d) $68°$ (e) $69°$

(3 pts) 7. C

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET | INDIVIDUAL TEST #2
University of North Dakota | Grades 11/12
February 20, 2023

School ______________ Team Name ______________

Calculators are **NOT** allowed. **Solutions** Student Name ______________

1. Let x be a real number. If $3^{x^4} = 81$, then x^2 is

   (a) -2 (b) 2 (c) 3 (d) 4 (e) not uniquely determined

   (2 pts) 1. B

   Solution: Observe that $3^{x^4} = 81 = 3^4$, thus we have $x^4 = 4$. Since $x^2 \geq 0$, we have $x^2 = 2$. Thus, the correct answer is (B).

2. The 25 cells of square of a 5 by 5 grid are filled with 1's, 2's and 3's, with each cell getting exactly one number, and so that no two cells touching each other either horizontally or vertically get the same number. The numbers in the outside cells are

1	2	3	1	2
2				3
1		?		2
3				1
2	3	1	2	3

   shown. What is the number in the center cell?

   (a) 1 (b) 2 (c) 3 (d) no number is possible (e) more than one is possible

   (3 pts) 2. C

   Solution: Let a_{ij} be the number in the cell at row i and column j. First observe that we must have $a_{24} = 2$ and $a_{44} = 3$. Then, proceeding one-by-one we see that $a_{23} = 1$, $a22 = 2$, $a_{32} = 2$, $a_{42} = 1$, $a_{43} = 2$, and $a_{34} = 1$. Hence, it must be that $a_{33} = 3$. The answer is (C).

3. For how many integers n is the fraction $\frac{6300}{n}$ the square of an integer?

   (a) 4 (b) 7 (c) 8 (d) 14 (e) 16

   (3 pts) 3. C

   Solution: Since $6300 = 2^2 \cdot 3^2 \cdot 5^2 \cdot 7$, the positive integer n could take one of the following values

$$7, \quad 2^2 \cdot 7, \quad 3^2 \cdot 7, \quad 5^2 \cdot 7, \quad 2^2 \cdot 3^2 \cdot 7, \quad 2^2 \cdot 5^2 \cdot 7, \quad 3^2 \cdot 5^2 \cdot 7, \quad 2^2 \cdot 3^2 \cdot 5^2 \cdot 7$$

   and thus there are 8 possible values for n. Hence, the answer is (C).

4. Let $g(k) = 2k^2$ for any positive integer k. Let $p \geq 3$ be a prime number. The number of positive integers dividing evenly into $g(g(p))$ is

   (a) 7 (b) 8 (c) 15 (d) 20 (e) depends on p

   (3 pts) 4. D

   Solution: Observe that $g(g(p)) = 2(2p^2)^2 = 2^3 p^4$. Thus, the positive divisors of $g(g(p))$ are all of the form $2^i p^j$ for integers $i = 0, 1, 2, 3$ and $j = 0, 1, 2, 3, 4$. Hence, $g(g(p))$ has $4 \times 5 = 20$ positive divisors. Thus, the answer is (D).

5. At a daycare with 100 children, each child can bring at most 10 toys (but could also bring fewer than 10 toys, including no toys at all). Without knowing how many toys each child will bring, what is the maximum value of n for which the following statement is true?

At least n children must bring the same number of toys.

(a) 9 (b) 10 (c) 11 (d) 12 (e) 100

(3 pts) 5. B

Solution: Each child can bring $0, 1, 2, \ldots, 10$ toys. Since there are 100 children, at least $\lceil 100/11 \rceil = \lceil 9.\overline{09} \rceil = 10$ must bring the same number of toys. This shows that $n \geq 10$. To show that $n = 10$ exactly, suppose that 10 children brought 0 toys each, 10 more children brought 1 toy each, and so on until the last 10 children brought 9 toys each. This accounts for all 100 children, and in this situation the statement is false for any $n > 10$. Therefore, the answer is (B).

6. Let m, n be positive integers such that $\frac{1}{m} + \frac{1}{n} = \frac{19}{94}$. The value of $m + n$ is equal to

(a) 19 (b) 180 (c) 345 (d) 475 (e) 570

(3 pts) 6. D

Solution: We may assume that $m \geq n$, hence

$$\frac{19n}{94} = \frac{n}{m} + 1 \in (1, 2] \iff n \in \left(\frac{94}{19}, \frac{188}{19}\right] \iff n \in \{5, 6, 7, 8, 9\}.$$

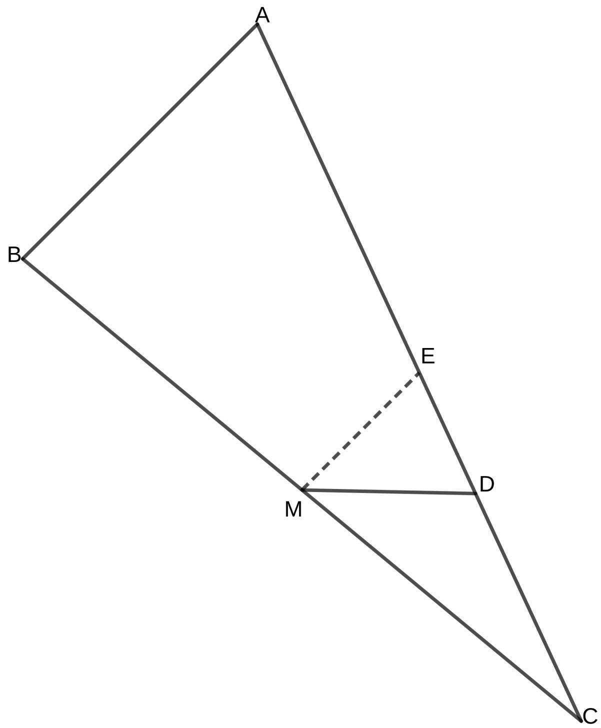
Only $n = 5$ is possible, which give $m = 470$, and thus $n + m = 475$. The answer is (D).

7. In a triangle $\triangle ABC$, we have the segment lengths $\ell(\overline{AB}) = 4$, $\ell(\overline{AC}) = 2\sqrt{23}$, the measure of the angle $\angle BAC = 46°$. D is a point on segment $\overline{AC}$ such that $\ell(\overline{AD}) = \sqrt{23} + 2$. And, M is the midpoint of $\overline{BC}$. The measure of angle $\angle ADM$ is:

(a) 65° (b) 66° (c) 67° (d) 68° (e) 69°

(3 pts) 7. C

Solution: Let E be the midpoint of $\overline{AC}$. $\overline{ME}$ is a midline in $\triangle ABC$, so $\overline{ME} \parallel \overline{AB}$ and $\ell(\overline{ME}) = \ell(\overline{AB})/2 = 2$. Thus $\angle MED = \angle BAC = 46°$. Meanwhile, $\ell(\overline{ED}) = \ell(\overline{AD}) - \ell(\overline{AE}) = \sqrt{23} + 2 - \sqrt{23} = 2$. Hence $\triangle MED$ is isosceles and the measure of $\angle ADM = \frac{180° - 46°}{2} = 67°$. The answer is (C).


A
B
E
D
M
C

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #3
Grades 11/12

School ______________

Team Name ______________

Calculators are allowed.
All answers should be exact or rounded to three significant figures.

Student Name ______________

1. How many four digit PINs can be created if the digits are strictly increasing? (2pts) 1. ________

2. What is the probability that if you flip a coin six times, there are an even number of tails? (3pts) 2. ________

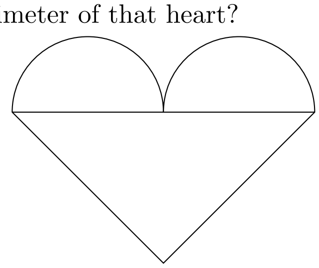
3. Find the angle in degrees between the minute and hour hands of a clock when the time is 8:30. (3pts) 3. ________

4. Alice and Beth can shovel a driveway in one hour. Working alone, it would take Alice 90 minutes. How long would it take Beth working alone? (3pts) 4. ________

5. A house has people with two legs, two eyes, and no tail; dogs with four legs, two eyes, and one tail; and birds with two legs, two eyes, and one tail. Combined, there are 20 legs, 16 eyes, and 5 tails. How many dogs are there? (3pts) 5. ________

6. How many positive integers evenly divide 2023? (3pts) 6. ________

7. We can make a heart by placing two semicircles of radius 1 atop a right triangle as below. What is the perimeter of that heart?



(3pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET | INDIVIDUAL TEST #3
University of North Dakota | Grades 11/12
February 20, 2023

SOLUTION KEY **SOLUTION KEY** **SOLUTION KEY**

School ______________ Team Name ______________

Calculators are allowed. Student Name ______________

All answers should be exact or rounded to three significant figures.

1. How many four digit PINs can be created if the digits are strictly increasing? (2pts) 1. ________ $\binom{10}{4} = 210$

2. What is the probability that if you flip a coin six times, there are an even number of tails?
Hint: $(\binom{6}{0} + \binom{6}{2} + \binom{6}{4} + \binom{6}{6})/2^6$ *(also, when you get to the last coin, there are either an even or an odd number of tails so far)* (3pts) 2. ________ $\frac{32}{64} = \frac{1}{2} = 0.5$

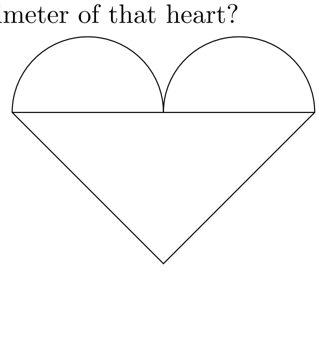
3. Find the angle in degrees between the minute and hour hands of a clock when the time is 8:30. (3pts) 3. ________ 75°

4. Alice and Beth can shovel a driveway in one hour. Working alone, it would take Alice 90 minutes. How long would it take Beth working alone?
Hint: Alice can shovel 2/3 of the driveway in one hour (3pts) 4. ________ 3 hours or 180 mins

5. A house has people with two legs, two eyes, and no tail; dogs with four legs, two eyes, and one tail; and birds with two legs, two eyes, and one tail. Combined, there are 20 legs, 16 eyes, and 5 tails. How many dogs are there?
Hint: 3 people, 2 dogs, 3 birds (3pts) 5. ________ 2

6. How many positive integers evenly divide 2023?
Hint: $7^i 17^j$, *where* $0 \le i \le 1$ *and* $0 \le j \le 2$ (3pts) 6. ________ 6

7. We can make a heart by placing two semicircles of radius 1 atop a right triangle as below. What is the perimeter of that heart?



(3pts) 7. ________ $2\pi + 4\sqrt{2} \approx 11.9$

SOLUTION KEY **SOLUTION KEY** **SOLUTION KEY**

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #4
Grades 11/12

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Student Name ______________

1. Let ABC be a right triangle with right angle at B. Draw the segment $\overline{\text{BD}}$ so that it bisects $\overline{\text{AC}}$. If $m\angle\text{BAD} = 52°$, what is $m\angle\text{BDC}$ in degrees?

(2 pts) 1. ________

2. A particular 12-hour digital clock displays the hour and minute of a day. Unfortunately, whenever it is supposed to display a 1, it mistakenly displays a 9. For example, when it is 1:16 PM the clock incorrectly shows 9:96 PM. What fraction of the day will the clock show the correct time?

(3 pts) 2. ________

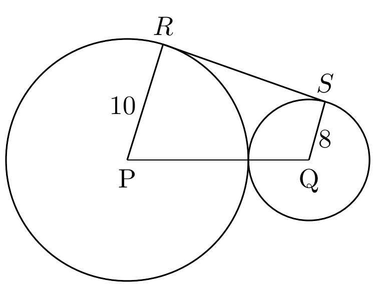
3. Solve the inequality $\sqrt{(x-2)^2} < |x|$. Express your solution in interval notation.

(3 pts) 3. ________

4. Given $\dfrac{1-\sin\theta}{1+\sin\theta} = 3$. What is the exact value of $2(\sec\theta - \tan\theta)^2$?

(3 pts) 4. ________

5. $\overline{\text{RS}}$ is a tangent to the circles with centers at P and Q as shown below (figure not to scale). Find $\overline{\text{RS}}$. Give your answer in the simplest form.



(3 pts) 5. ________

6. Solve for x:

$$\log_5(\log_2(\log_3(2x-3))) = 0$$

(3 pts) 6. ________

7. The curve $y = x^4 - 8x^3 + 9x^2 + 20x + 2$ and the line $y = 2x + 1$ intersect at four distinct points in the real xy-plane. Find the average value of the y-coordinates of the intersection points.

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #4
Grades 11/12

School ______________ Team Name ______________

Calculators are **NOT** allowed. **Key** Student Name ______________

1. Let ABC be a right triangle with right angle at B. Draw the segment $\overline{\text{BD}}$ so that it bisects $\overline{\text{AC}}$. If $m\angle\text{BAD} = 52°$, what is $m\angle\text{BDC}$ in degrees?

(2 pts) 1. 104

2. A particular 12-hour digital clock displays the hour and minute of a day. Unfortunately, whenever it is supposed to display a 1, it mistakenly displays a 9. For example, when it is 1:16 PM the clock incorrectly shows 9:96 PM. What fraction of the day will the clock show the correct time?

(3 pts) 2. $\frac{1}{2}$

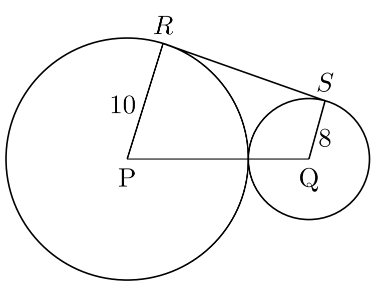
3. Solve the inequality $\sqrt{(x-2)^2} < |x|$. Express your solution in interval notation.

(3 pts) 3. $(1, \infty)$

4. Given $\dfrac{1-\sin\theta}{1+\sin\theta} = 3$. What is the exact value of $2(\sec\theta - \tan\theta)^2$?

(3 pts) 4. 6

5. $\overline{\text{RS}}$ is a tangent to the circles with centers at P and Q as shown below (figure not to scale). Find $\overline{\text{RS}}$. Give your answer in the simplest form.



(3 pts) 5. $8\sqrt{5}$

6. Solve for x:

$$\log_5(\log_2(\log_3(2x-3))) = 0$$

(3 pts) 6. 6

7. The curve $y = x^4 - 8x^3 + 9x^2 + 20x + 2$ and the line $y = 2x + 1$ intersect at four distinct points in the real xy-plane. Find the average value of the y-coordinates of the intersection points.

(3 pts) 7. 5

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

INDIVIDUAL TEST #4
Grades 11/12

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Solutions

Student Name ______________

1. Let ABC be a right triangle with right angle at B. Draw the segment $\overline{\text{BD}}$ so that it bisects $\overline{\text{AC}}$. If $m\angle\text{BAD} = 52°$, what is $m\angle\text{BDC}$ in degrees?

(2 pts) 1. 104

Solution: From the right triangle, $m\angle\text{BCD} = 38°$. If we circumscribe a circle around the triangle with diameter $\overline{\text{AC}}$, then the $\overline{\text{AD}} = \overline{\text{BD}} = \overline{\text{DC}}$ are all radii of the circle. From $\triangle\text{BDC}$, $m\angle\text{CBD} = \text{m}\angle\text{BCD} = 38°$. Therefor $\angle\text{BDC} = 180 - (38 + 38) = 180 - 76 = 104°$.

2. A particular 12-hour digital clock displays the hour and minute of a day. Unfortunately, whenever it is supposed to display a 1, it mistakenly displays a 9. For example, when it is 1:16 PM the clock incorrectly shows 9:96 PM. What fraction of the day will the clock show the correct time?

(3 pts) 2. $\frac{1}{2}$

Solution: 1 is in the hourly position for 4 full hours of the day (1-2, 10-11, 11-12 and 12-1) and 1 is in the minutes position at following minutes during each hour: $1, 10-19, 21, 31, 41, 51$. That is 15 minutes each hour. Since we have already counted 4 full hours, there will be an additional 8*15 minutes =120 minutes (or 2 hours) when the clock is incorrect. That means the clock is incorrect for 6 hours out of 12 or 12 hours out of 24 or $\frac{1}{2}$ of the day.

3. Solve the inequality $\sqrt{(x-2)^2} < |x|$. Express your solution in interval notation.

(3 pts) 3. $(1, \infty)$

Solution: We need to solve $|x-2| < |x|$. If $x < 0$, the equation becomes $2-x < -x$ which has no solution. If $0 \leq x < 2$, we can rewrite the equation as $2-x < x$ which gives $x > 1$. If $x \geq 2$, the equation is $x - 2 < x$ which is true for all values of x. Therefore the solution is given by $(1, \infty)$.

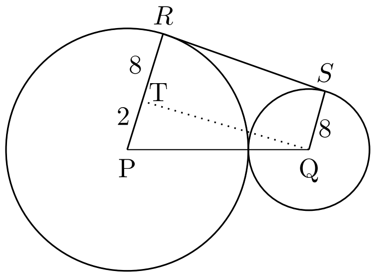
4. Given $\dfrac{1-\sin\theta}{1+\sin\theta} = 3$. What is the exact value of $2(\sec\theta - \tan\theta)^2$?

(3 pts) 4. 6

Solution:

$$2(\sec\theta - \tan\theta)^2 = 2\left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = 2\frac{(1-\sin\theta)^2}{\cos^2\theta} = 2\frac{(1-\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)} = 2\frac{1-\sin\theta}{1+\sin\theta} = 2(3) = 6$$

5. $\overline{\text{RS}}$ is a tangent to the circles with centers at P and Q as shown below (figure not to scale). Find $\overline{\text{RS}}$. Give your answer in the simplest form.



(3 pts) 5. $8\sqrt{5}$

Solution: Since $\overline{\text{RS}}$ is a tangent to both circles, $\overline{\text{RS}} \perp \overline{\text{RP}}$ and $\overline{\text{RS}} \perp \overline{\text{SQ}}$. Draw $\overline{\text{QT}} \perp \overline{\text{RP}}$ and let $\overline{\text{RS}} = \overline{\text{QT}} = x$. Note that $\overline{\text{RT}} = \overline{\text{SQ}} = 8$ and so $\overline{\text{TP}} = 2$. From the right $\triangle$PTQ, we have $\overline{\text{PQ}}^2 = x^2 + 4$ and since $\overline{\text{PQ}} = 10 + 8 = 18$, we get $x^2 = 324 - 4 = 320 \Rightarrow x = \sqrt{320} = 8\sqrt{5}$.

6. Solve for x:

$$\log_5(\log_2(\log_3(2x - 3))) = 0$$

(3 pts) 6. 6

Solution: Using definition of logs, $\log_2(\log_3(2x - 3)) = 5^0 = 1 \Rightarrow \log_3(2x - 3) = 2^1 = 2 \Rightarrow 2x - 3 = 3^2 \Rightarrow 2x = 12 \Rightarrow x = 6$

7. The curve $y = x^4 - 8x^3 + 9x^2 + 20x + 2$ and the line $y = 2x + 1$ intersect at four distinct points in the real xy-plane. Find the average value of the y-coordinates of the intersection points.

(3 pts) 7. 5

Solution: The points of intersection are given by $x^4 - 8x^3 + 9x^2 + 20x + 2 = 2x + 1$ or $x^4 - 8x^3 + 9x^2 + 18x + 1 = 0$. If the points of intersection are given by (x_i, y_i) for $i = 1, \cdots, 4$. Then $\sum_{i=1}^{4} x_i$ is the sum of roots of the polynomial and $= 8$. Since all four points lie on the line $y = 2x + 1$, we can write $\sum_{i=1}^{4} y_i = 2\sum_{i=1}^{4} x_i + 4 = 20$ and the average value of the y-coordinates is given by $\bar{y} = \frac{\sum_{i=1}^{4} y_i}{4} = \frac{20}{4} = 5$

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

TEAM TEST #2
Grades 11/12

School ______________ Team Name ______________

Calculators are **NOT** allowed.

1. Suppose $x^2 - (x-3)^2 = 9009$. Determine x.

(20 pts) 1. ________

2. Simplify the following expression as much as possible:

$$\frac{1}{\log_8(10)} + \frac{3}{\log_5(10)}.$$

(20 pts) 2. ________

3. Find the acute angle in degrees between the hour and minute hands on an analog clock at the time 11:05.

(20 pts) 3. ________

4. What is the smallest integer n such that $\dfrac{2023}{n}$ is a perfect square?

(20 pts) 4. ________

5. Find the derivative of $y = \ln(x)$ at $x = 0.05$.

(20 pts) 5. ________

6. Determine the radius of the circle given by $x^2 + 8x + y^2 - 2y = 424$.

(20 pts) 6. ________

7. What is the period in radians of $y = \sin\left(\dfrac{x}{5}\right)\cos(3x)$?

(20 pts) 7. ________

8. Evaluate

$$\sum_{k=1}^{\infty} \frac{2}{n^2 + 2n}.$$

(20 pts) 8. ________

9. Assume a closed box has dimensions $2 \times 3 \times 5$. Find the ratio of the surface area to the volume of the box in lowest terms.

(20 pts) 9. ________

10. Suppose $\cos\left(\dfrac{\theta}{2}\right) = \dfrac{\sqrt{5}}{3}$. Determine $\sec(\theta)$.

(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

TEAM TEST #2
Grades 11/12

School ______________ **Key** Team Name ______________

Calculators are **NOT** allowed.

1. Suppose $x^2 - (x-3)^2 = 9009$. Determine x.

(20 pts) 1. 1503 or $x = 1503$

2. Simplify the following expression as much as possible:

$$\frac{1}{\log_8(10)} + \frac{3}{\log_5(10)}.$$

(20 pts) 2. 3

3. Find the acute angle in degrees between the hour and minute hands on an analog clock at the time 11:05.

(20 pts) 3. $\frac{115}{2}$ or 57.5

4. What is the smallest integer n such that $\frac{2023}{n}$ is a perfect square?

(20 pts) 4. 7 or $n = 7$

5. Find the derivative of $y = \ln(x)$ at $x = 0.05$.

(20 pts) 5. 20

6. Determine the radius of the circle given by $x^2 + 8x + y^2 - 2y = 424$.

(20 pts) 6. 21

7. What is the period in radians of $y = \sin\left(\frac{x}{5}\right)\cos(3x)$?

(20 pts) 7. 5π

8. Evaluate

$$\sum_{k=1}^{\infty} \frac{2}{n^2 + 2n}.$$

(20 pts) 8. $\frac{3}{2}$ or 1.5

9. Assume a closed box has dimensions $2 \times 3 \times 5$. Find the ratio of the surface area to the volume of the box in lowest terms.

(20 pts) 9. $\frac{31}{15}$

10. Suppose $\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{5}}{3}$. Determine $\sec(\theta)$.

(20 pts) 10. 9

Math Track Meet 2023
Team Test #2
Grades 11/12
Solutions

1. We have

$$\begin{aligned} x^2-(x-3)^2 &= 9009 \\ x^2-x^2+6x-9 &= 9009 \\ 6x &= 9018 \\ x &= \frac{9018}{6} = 1503 \end{aligned}$$

2. Observe

$$\frac{1}{\log_8(10)}+\frac{3}{\log_5(10)} = \frac{1}{\frac{\log(10)}{\log(8)}}+\frac{3}{\frac{\log(10)}{\log(5)}} = \log(8)+3\log(5) = \log(8\cdot 5^3) = \log(1000) = 3.$$

3. Measure degrees clockwise with noon as reference point. The minute hand moves $\frac{360^\circ}{60} = 6^\circ$ per minute. The hour hand moves $\frac{360^\circ}{12} = 30^\circ$ per hour. At 11:05, the minute hand has moved 30° or 330°. At 11:00, the hour hand has moved to -30°. In the following 5 minutes, the hour hand sweeps an additional $\frac{5}{60}(30^\circ) = 2.5^\circ$. So the desired acute angle is $30^\circ+30^\circ-2.5^\circ = 57.5^\circ$.

4. Observe $2023 = 7\cdot 17^2$. Taking $n=7$ gives the perfect square $\frac{2023}{7} = \frac{7\cdot 17^2}{7} = 17^2$. Thus, $n=7$.

5. The derivative of $y=\ln(x)$ is $y' = \frac{1}{x}$. At $x = 0.05 = \frac{1}{20}$, $y'(1/20) = \frac{1}{1/20} = 20$.

6. We have

$$\begin{aligned} x^2+8x+y^2-2y &= 424 \\ x^2+8x+16+y^2-2y+1 &= 424+16+1 \\ (x+4)^2+(y-1)^2 &= 441. \end{aligned}$$

Thus the radius is $r = \sqrt{441} = 21$.

7. The period of $y = \sin\left(\frac{x}{5}\right)\cos(3x)$ is 5π.

8. We have

$$\begin{aligned} \sum_{k=1}^{\infty}\frac{2}{n^2+2n} &= \sum_{k=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right) \\ &= \frac{1}{1}+\frac{1}{2}+\sum_{k=3}^{\infty}\frac{1}{n}-\sum_{k=1}^{\infty}\frac{1}{n+2} \\ &= \frac{1}{1}+\frac{1}{2}+\sum_{k=1}^{\infty}\frac{1}{n+2}-\sum_{k=1}^{\infty}\frac{1}{n+2} \\ &= \frac{3}{2} \end{aligned}$$

9. The ratio of surface area to volume for a box of dimenions $a \times b \times c$ is $\frac{2ab+2ac+2bc}{abc}$. Taking $a = 2$, $b = 3$, and $c = 5$ yields a ratio of

$$\frac{2ab + 2ac + 2bc}{abc} = \frac{2(2)(3) + 2(2)(5) + 2(3)(5)}{2(3)(5)} = \frac{12 + 20 + 30}{30} = \frac{62}{30} = \frac{31}{15}.$$

10. The half-angle identity $\cos(\theta/2) = \pm\sqrt{\frac{1+\cos\theta}{2}}$ implies $\cos(\theta) = -1 + 2 \cdot \cos^2\left(\frac{\theta}{2}\right)$. It follows that

$$\cos(\theta) = -1 + 2\left(\frac{\sqrt{5}}{3}\right)^2 = -1 + \frac{10}{9} = \frac{1}{9}.$$

Thus $\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{1/9} = 9$.

UND MATHEMATICS TRACK MEET
University of North Dakota
February 20, 2023

TEAM TEST #2
Grades 11/12

School ______________

Team Name ______________

Calculators are **NOT** allowed.

1. Suppose $x^2 - (x-3)^2 = 9009$. Determine x.

(20 pts) 1. ________

2. Simplify the following expression as much as possible:

$$\frac{1}{\log_8(10)} + \frac{3}{\log_5(10)}.$$

(20 pts) 2. ________

3. Find the acute angle in degrees between the hour and minute hands on an analog clock at the time 11:05.

(20 pts) 3. ________

4. What is the smallest integer n such that $\dfrac{2023}{n}$ is a perfect square?

(20 pts) 4. ________

5. Find the derivative of $y = \ln(x)$ at $x = 0.05$.

(20 pts) 5. ________

6. Determine the radius of the circle given by $x^2 + 8x + y^2 - 2y = 424$.

(20 pts) 6. ________

7. What is the period in radians of $y = \sin\left(\dfrac{x}{5}\right)\cos(3x)$?

(20 pts) 7. ________

8. Evaluate

$$\sum_{k=1}^{\infty} \frac{2}{n^2+2n}.$$

(20 pts) 8. ________

9. Assume a closed box has dimensions $2 \times 3 \times 5$. Find the ratio of the surface area to the volume of the box in lowest terms.

(20 pts) 9. ________

10. Suppose $\cos\left(\dfrac{\theta}{2}\right) = \dfrac{\sqrt{5}}{3}$. Determine $\sec(\theta)$.

(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
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TEAM TEST #2
Grades 11/12

School ______________ **Key** Team Name ______________

Calculators are **NOT** allowed.

1. Suppose $x^2 - (x-3)^2 = 9009$. Determine x.

(20 pts) 1. 1503 or $x = 1503$

2. Simplify the following expression as much as possible:

$$\frac{1}{\log_8(10)} + \frac{3}{\log_5(10)}.$$

(20 pts) 2. 3

3. Find the acute angle in degrees between the hour and minute hands on an analog clock at the time 11:05.

(20 pts) 3. $\frac{115}{2}$ or 57.5

4. What is the smallest integer n such that $\frac{2023}{n}$ is a perfect square?

(20 pts) 4. 7 or $n = 7$

5. Find the derivative of $y = \ln(x)$ at $x = 0.05$.

(20 pts) 5. 20

6. Determine the radius of the circle given by $x^2 + 8x + y^2 - 2y = 424$.

(20 pts) 6. 21

7. What is the period in radians of $y = \sin\left(\frac{x}{5}\right)\cos(3x)$?

(20 pts) 7. 5π

8. Evaluate

$$\sum_{k=1}^{\infty} \frac{2}{n^2 + 2n}.$$

(20 pts) 8. $\frac{3}{2}$ or 1.5

9. Assume a closed box has dimensions $2 \times 3 \times 5$. Find the ratio of the surface area to the volume of the box in lowest terms.

(20 pts) 9. $\frac{31}{15}$

10. Suppose $\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{5}}{3}$. Determine $\sec(\theta)$.

(20 pts) 10. 9

Math Track Meet 2023
Team Test #2
Grades 11/12
Solutions

1. We have

$$\begin{aligned} x^2-(x-3)^2 &= 9009 \\ x^2-x^2+6x-9 &= 9009 \\ 6x &= 9018 \\ x &= \frac{9018}{6} = 1503 \end{aligned}$$

2. Observe

$$\frac{1}{\log_8(10)} + \frac{3}{\log_5(10)} = \frac{1}{\frac{\log(10)}{\log(8)}} + \frac{3}{\frac{\log(10)}{\log(5)}} = \log(8) + 3\log(5) = \log(8\cdot 5^3) = \log(1000) = 3.$$

3. Measure degrees clockwise with noon as reference point. The minute hand moves $\frac{360°}{60} = 6°$ per minute. The hour hand moves $\frac{360°}{12} = 30°$ per hour. At 11:05, the minute hand has moved $30°$ or $330°$. At 11:00, the hour hand has moved to $-30°$. In the following 5 minutes, the hour hand sweeps an additional $\frac{5}{60}(30°) = 2.5°$. So the desired acute angle is $30° + 30° - 2.5° = 57.5°$.

4. Observe $2023 = 7\cdot 17^2$. Taking $n = 7$ gives the perfect square $\frac{2023}{7} = \frac{7\cdot 17^2}{7} = 17^2$. Thus, $n = 7$.

5. The derivative of $y = \ln(x)$ is $y' = \frac{1}{x}$. At $x = 0.05 = \frac{1}{20}$, $y'(1/20) = \frac{1}{1/20} = 20$.

6. We have

$$\begin{aligned} x^2+8x+y^2-2y &= 424 \\ x^2+8x+16+y^2-2y+1 &= 424+16+1 \\ (x+4)^2+(y-1)^2 &= 441. \end{aligned}$$

Thus the radius is $r = \sqrt{441} = 21$.

7. The period of $y = \sin\left(\frac{x}{5}\right)\cos(3x)$ is 5π.

8. We have

$$\begin{aligned} \sum_{k=1}^{\infty} \frac{2}{n^2+2n} &= \sum_{k=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+2}\right) \\ &= \frac{1}{1} + \frac{1}{2} + \sum_{k=3}^{\infty}\frac{1}{n} - \sum_{k=1}^{\infty}\frac{1}{n+2} \\ &= \frac{1}{1} + \frac{1}{2} + \sum_{k=1}^{\infty}\frac{1}{n+2} - \sum_{k=1}^{\infty}\frac{1}{n+2} \\ &= \frac{3}{2} \end{aligned}$$

9. The ratio of surface area to volume for a box of dimenions $a \times b \times c$ is $\frac{2ab+2ac+2bc}{abc}$. Taking $a = 2$, $b = 3$, and $c = 5$ yields a ratio of

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10. The half-angle identity $\cos(\theta/2) = \pm\sqrt{\frac{1+\cos\theta}{2}}$ implies $\cos(\theta) = -1 + 2 \cdot \cos^2\left(\frac{\theta}{2}\right)$. It follows that

$$\cos(\theta) = -1 + 2\left(\frac{\sqrt{5}}{3}\right)^2 = -1 + \frac{10}{9} = \frac{1}{9}.$$

Thus $\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{1}{1/9} = 9$.