2022 Math Track Meet

University of North Dakota

February 21, 2022

Contents

School _____   Team Name _____

Calculators are allowed.   Student Name _____

1. Find and simplify

$$\frac{5}{6} - \frac{4}{5} + \frac{3}{4} - \frac{2}{3}$$

Write your answer as a simple fraction where the numerator and denominator have no common factors.

(2 pts) 1. _____

2. Each student in a class has a dog, a cat, or both a dog and a cat. Seventeen students have a dog (with or without a cat), and 10 have a cat (with or without a dog). If 4 students have both a dog and a cat, how many students are in the class?

(3 pts) 2. _____

3. A recipe calls for $1\frac{1}{3}$ cups of sugar and $2\frac{1}{2}$ cups of flour. If we increase the recipe to use 2 cups of sugar, how many cups of flour should we use? Write your answer as a mixed number where the numerator and the denominator of the fraction have no common factors.

(3 pts) 3. _____

4. A square has sides of length 6 inches. A circle has a diameter of 7 inches. How much longer (in inches) is the perimeter of the square than the circumference of the circle? Use 3.14 for pi, and write your answer to the hundredth place.

(3 pts) 4. _____

5. What is the only prime factor of 2022 which is greater than 100?

(3 pts) 5. _____

6. Four students take a quiz. The highest score is 10, and the lowest is 4. If the median score is 8, what is the mean (average) score?

(3 pts) 6. _____

7. A triangular prism has right triangular faces with sides of length 10 cm, 8 cm, and 6 cm. The height of the prism is 4 cm. What is the volume of the prism (in cm^3)?

(3 pts) 7. _____

2

TOTAL POINTS _____

School ―――――――

Team Name ―――――――

Calculators are NOT allowed.

Student Name ―――――――

1. Evaluate

$$16 - 2\left|3 - (-5)\right| \div 7 - 4\left(2^2 + 1\right)$$

(2 pts) 1. ―――――

2. Solve for x and put the answer in interval form.

$$|5 - 3x| \geq 1$$

(3 pts) 2. ―――――

3. Simplify the expression

$$(-3)^0 + (-2)^3 + 2^3 + 3^2$$

(3 pts) 3. ―――――

4. Muriel worked for Mrs. Grateful for $\dfrac{2}{3}$ of an hour on Tuesday and on Wednesday for 1 and $\dfrac{1}{3}$ hours and on Friday for 4 hours. How many hours (in fraction form) did she work total? Answer must be in fraction form, not decimal.

(3 pts) 4. ―――――

5. There are 25 students in Mary's Ceramic class. Of those students, 10 of them have blue eyes. What percent of students do not have blue eyes?

(3 pts) 5. ―――――

6. Simplify

$$\frac{4x^3 y^5}{8x^7 y^{-2}}$$

(3 pts) 6. ―――――

7. Find the y-intercept for

$$f(x) = x^2 - 6x + 9$$

(3 pts) 7. ―――――

md

TOTAL POINTS ―――――

School _____

Calculators are allowed.

1. What is the prime factorization of 2,022?

   (2 pts) 1. _____

2. The *Tower of Hanoi* is a game involving 3 pegs, one of which has disks stacked in descending order. The goal is to move every disk to another peg, but you can only move one at a time, and bigger disks cannot go on top of smaller disks. What is the minimum number of moves to move 3 discs from one peg to another under these conditions?
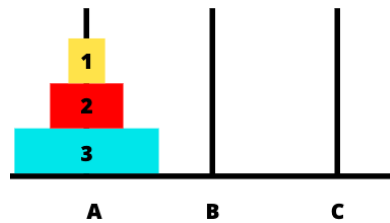
   (3 pts) 2. _____

3. When $(2,022)^{23}$ is completely multiplied out, what will the number be in the ones place?

   (3 pts) 3. _____

4. Mark has a large rock collection and wants to assign a unique 4-digit code to distinguish each rock using only the digits 1, 2, 3, and 4. For his numbering system, he wants each number to *only appear once* in each code (i.e., 1234 is valid, but 1122 is not). What is the maximum number of rocks that can be assigned unique code numbers?

   (3 pts) 4. _____

5. At a party with 10 people, each pair had a one-on-one conversation at some point throughout the night. If each pair interacted only once each, how many total one-on-one conversation were had?

   (3 pts) 5. _____

6. How many unique whole numbers less than 2,022 can be made from the set of digits $\{2, 4, 6, 8\}$? (It *is* okay to have repeating digits here.)

   (3 pts) 6. _____

7. A zoo has 60 different exhibits. Some of the exhibits hold 4 animals, and some hold only 2. All of the exhibits are at maximum capacity when there are 160 total animals. How many of the enclosures hold four animals?

   (3 pts) 7. _____

lm

TOTAL POINTS _____

School _____

Team Name _____

Calculators are NOT allowed.

Student Name _____

1. If one-third of a certain number is added to 7 and the sum is 19, what is that number?

   (2 pts) 1. _____

2. Of all the goats on the goat farm, 30% of them are pure white. If there are 18 white goats on the goat farm, how many goats in all are there on the goat farm?

   (3 pts) 2. _____

3. Liat has $\frac{3}{4}$ cup of orange juice. Sumin has $\frac{1}{3}$ less orange juice than Liat. How much orange juice does Sumin have? Remember to use proper label with your answer.

   (3 pts) 3. _____

4. There are 8 kids in the 8th grade Math Club. They want to elect co-presidents to take care of the Math Club's business. In how many ways can they elect the two officers called co-presidents?

   (3 pts) 4. _____

5. Jonah's favorite cookie recipe calls for 3 cups of flour. Jonah only has $\frac{3}{4}$ cup flour but he still wants to make a small batch. If his recipe also calls for 4 cups of sugar, how much sugar should Jonah use? Be sure your answer has the proper label.

   (3 pts) 5. _____

6. Two standard six-sided dice are thrown, and the dice are considered fair. What is the probability that the product of the two numbers facing up is odd?

   (3 pts) 6. _____

7. A large industrial container is cylindrical shaped with a radius of 3 ft and a height of 4 ft. What is the total surface area of the container if the top is included? Do not convert pi to a number. Be sure to use the proper label in your answer.

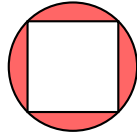   (3 pts) 7. _____

ch

TOTAL POINTS _____

School _____   Team Name _____

Calculators are allowed.

1. A square of side length 2 is inscribed in a circle. Find the area of the shaded region. Round your answer to the hundredth place.

(20 pts) 1. _____

2. The area of a triangle is 90. If the height of the triangle is one fifth its base, how long is the base?

(20 pts) 2. _____

3. Solve the system of equations. Write your answer in the form (x, y).
$$\begin{cases} 3x + 4y &= 1 \\ 5x + 6y &= 5 \end{cases}$$

(20 pts) 3. _____

4. Suppose $a + b = 7$ and $ab = 3$. What is $a^2 + b^2$?

(20 pts) 4. _____

5. In a class your grade is determined by four equal weight exams. Suppose your grade was 84% and the scores of your last three exams were 75%, 81%, and 93%. What score did you receive on the first exam?

(20 pts) 5. _____

6. There are 39 butterflies in one section of a flower garden. This represents 6% of the population of all butterflies in the garden. How many butterflies are there in the entire garden?

(20 pts) 6. _____

7. A bacterial culture doubles every 20 minutes. There are 22,528 bacteria at noon. How many bacteria were there at 10 am earlier that same day?

(20 pts) 7. _____

8. What is the greatest common divisor of 360 and 525?

(20 pts) 8. _____

9. What is the difference between the least positive odd number divisible by three distinct primes and the least positive even number divisible by three distinct primes?

(20 pts) 9. _____

10. Suppose the price of an item is decreased by 30% and then later the new price is increased by 50%. Compared to the original price, what percentage increase does the final price represent?

(20 pts) 10. _____

ab   TOTAL POINTS _____

School _____   Team Name _____

Calculators are NOT allowed.

1. How many of the first 1000 positive integers are multiples of both 4 and 5 but not of 6?

(20 pts) 1. _____

2. If a radius of a circle whose area is 36π cm^2 equals the width of a rectangle, and the diameter of the circle is half the length of the rectangle, then what is the perimeter of the rectangle (in cm)?

(20 pts) 2. _____

3. A coin jar has 100 pennies, 200 nickels, 300 dimes, and 400 quarters in it. What is the total value (in dollars) of all the coins in the jar?

(20 pts) 3. _____

4. A box contains igneous, metamorphic and sedimentary rocks. $\dfrac{1}{3}$ of the rocks are igneous, 60 are metamorphic, and 40% are sedimentary. How many total rocks are in the box?

(20 pts) 4. _____

5. If I start with $100 in my bank account and then increase my money by 20% in the first year, again increase my money by 20% in the second year, and then lose 50% of my money in the third year, how much money do I have left?

(20 pts) 5. _____

6. Evaluate
$$\left(\frac{1}{5} + \frac{5}{1} + \frac{5}{20} + \frac{20}{5}\right) - \left(\frac{1}{4} + \frac{4}{1} + \frac{2}{10} + \frac{10}{2}\right)$$

(20 pts) 6. _____

7. Sam cleared 12 levels on his computer game in 30 minutes. If she continues at this rate, how many minutes will it take her to clear 40 levels?

(20 pts) 7. _____

8. Consider the sequence of numbers 11, 28, 45 . . . , in which each number is 17 more than its predecessor. What is the 100th number in the sequence?

(20 pts) 8. _____

9. If $-400 \le x \le -300$ and $400 \le y \le 1200$. What is the largest value of $\dfrac{y}{x}$?

(20 pts) 9. _____

10. A drawer has 2 blue socks and 4 red socks. Another drawer has 6 blue socks and 8 red socks. If you take out a sock from each drawer, what is the probability that you will get a matching pair? Input your answer as a fraction in simplest form.

(20 pts) 10. _____

rp   TOTAL POINTS _____

UND MATHEMATICS TRACK MEET   INDIVIDUAL TEST #1
University of North Dakota   Grades 7/8
February 21, 2022

School _____   Team Name _____

Calculators are allowed.   **Key**   Student Name _____

1. Find and simplify

$$\frac{5}{6} - \frac{4}{5} + \frac{3}{4} - \frac{2}{3}$$

   Write your answer as a simple fraction where the numerator and denominator have no common factors.

   (2 pts) 1. $\dfrac{7}{60}$

2. Each student in a class has a dog, a cat, or both a dog and a cat. Seventeen students have a dog (with or without a cat), and 10 have a cat (with or without a dog). If 4 students have both a dog and a cat, how many students are in the class?

   (3 pts) 2. $\underline{\quad 23 \quad}$

3. A recipe calls for $1\frac{1}{3}$ cups of sugar and $2\frac{1}{2}$ cups of flour. If we increase the recipe to use 2 cups of sugar, how many cups of flour should we use? Write your answer as a mixed number where the numerator and the denominator of the fraction have no common factors.

   (3 pts) 3. $3\dfrac{3}{4}$

4. A square has sides of length 6 inches. A circle has a diameter of 7 inches. How much longer (in inches) is the perimeter of the square than the circumference of the circle? Use 3.14 for pi, and write your answer to the hundredth place.

   (3 pts) 4. $\underline{\quad 2.02 \quad}$

5. What is the only prime factor of 2022 which is greater than 100?

   (3 pts) 5. $\underline{\quad 337 \quad}$

6. Four students take a quiz. The highest score is 10, and the lowest is 4. If the median score is 8, what is the mean (average) score?

   (3 pts) 6. $\text{7.5 or } 7\dfrac{1}{2}$

7. A triangular prism has right triangular faces with sides of length 10 cm, 8 cm, and 6 cm. The height of the prism is 4 cm. What is the volume of the prism (in cm^3)?

   (3 pts) 7. $\underline{\quad 96 \quad}$

School ⸺⸺⸺⸺

Calculators are NOT allowed.　　**Key**

Team Name ⸺⸺⸺⸺

Student Name ⸺⸺⸺⸺

1. Evaluate

$$16 - 2\left|3 - (-5)\right| \div 7 - 4\left(2^2 + 1\right)$$

(2 pts) 1.　$-\dfrac{44}{7}$

2. Solve for x and put the answer in interval form.

$$\left|5 - 3x\right| \geq 1$$

(3 pts) 2.　$\left(-\infty, \dfrac{4}{3}\right] \cup [2, \infty)$

3. Simplify the expression

$$(-3)^0 + (-2)^3 + 2^3 + 3^2$$

(3 pts) 3.　10

4. Muriel worked for Mrs. Grateful for $\dfrac{2}{3}$ of an hour on Tuesday and on Wednesday for 1 and $\dfrac{1}{3}$ hours and on Friday for 4 hours. How many hours (in fraction form) did she work total? Answer must be in fraction form, not decimal.

(3 pts) 4.　6

5. There are 25 students in Mary's Ceramic class. Of those students, 10 of them have blue eyes. What percent of students do not have blue eyes?

(3 pts) 5.　60%

6. Simplify

$$\frac{4x^3 y^5}{8x^7 y^{-2}}$$

(3 pts) 6.　$\dfrac{y^7}{2x^4}$

7. Find the y-intercept for

$$f(x) = x^2 - 6x + 9$$

(3 pts) 7.　9

School _____   Team Name _____

Calculators are allowed.   **Key**   Student Name _____

1. What is the prime factorization of 2,022?

   (2 pts) 1. $\underline{2 \cdot 3 \cdot 337}$

2. The *Tower of Hanoi* is a game involving 3 pegs, one of which has disks stacked in descending order. The goal is to move every disk to another peg, but you can only move one at a time, and bigger disks cannot go on top of smaller disks. What is the minimum number of moves to move 3 discs from one peg to another under these conditions?
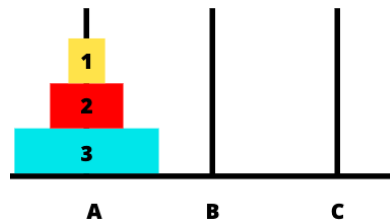
   (3 pts) 2. ___7___

3. When $(2,022)^{23}$ is completely multiplied out, what will the number be in the ones place?

   (3 pts) 3. ___8___

4. Mark has a large rock collection and wants to assign a unique 4-digit code to distinguish each rock using only the digits 1, 2, 3, and 4. For his numbering system, he wants each number to *only appear once* in each code (i.e., 1234 is valid, but 1122 is not). What is the maximum number of rocks that can be assigned unique code numbers?

   (3 pts) 4. ___24___

5. At a party with 10 people, each pair had a one-on-one conversation at some point throughout the night. If each pair interacted only once each, how many total one-on-one conversation were had?

   (3 pts) 5. ___45___

6. How many unique whole numbers less than 2,022 can be made from the set of digits $\{2, 4, 6, 8\}$? (It *is* okay to have repeating digits here.)

   (3 pts) 6. ___84___

7. A zoo has 60 different exhibits. Some of the exhibits hold 4 animals, and some hold only 2. All of the exhibits are at maximum capacity when there are 160 total animals. How many of the enclosures hold four animals?

   (3 pts) 7. ___20___

School _____   Team Name _____

Calculators are NOT allowed.   **Key**   Student Name _____

1. If one-third of a certain number is added to 7 and the sum is 19, what is that number?

   (2 pts) 1. ___36___

2. Of all the goats on the goat farm, 30% of them are pure white. If there are 18 white goats on the goat farm, how many goats in all are there on the goat farm?

   (3 pts) 2. ___60___

3. Liat has $\frac{3}{4}$ cup of orange juice. Sumin has $\frac{1}{3}$ less orange juice than Liat. How much orange juice does Sumin have? Remember to use proper label with your answer.

   (3 pts) 3. $\frac{1}{2}$ cup

4. There are 8 kids in the 8th grade Math Club. They want to elect co-presidents to take care of the Math Club's business. In how many ways can they elect the two officers called co-presidents?

   (3 pts) 4. ___28___

5. Jonah's favorite cookie recipe calls for 3 cups of flour. Jonah only has $\frac{3}{4}$ cup flour but he still wants to make a small batch. If his recipe also calls for 4 cups of sugar, how much sugar should Jonah use? Be sure your answer has the proper label.

   (3 pts) 5. ___1 cup___

6. Two standard six-sided dice are thrown, and the dice are considered fair. What is the probability that the product of the two numbers facing up is odd?

   (3 pts) 6. $\frac{1}{4}$

7. A large industrial container is cylindrical shaped with a radius of 3 ft and a height of 4 ft. What is the total surface area of the container if the top is included? Do not convert pi to a number. Be sure to use the proper label in your answer.
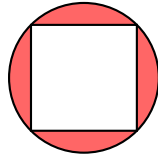
   (3 pts) 7. 42π sq. ft.

School ——————   **Key**   Team Name ——————

Calculators are allowed.

1. A square of side length 2 is inscribed in a circle. Find the area of the shaded region. Round your answer to the hundredth place.

(20 pts) 1. __2.28__

2. The area of a triangle is 90. If the height of the triangle is one fifth its base, how long is the base?

(20 pts) 2. __30__

3. Solve the system of equations. Write your answer in the form (x, y).
$$\begin{cases} 3x + 4y = 1 \\ 5x + 6y = 5 \end{cases}$$

(20 pts) 3. __(7, −5)__

4. Suppose $a + b = 7$ and $ab = 3$. What is $a^2 + b^2$?

(20 pts) 4. __43__

5. In a class your grade is determined by four equal weight exams. Suppose your grade was 84% and the scores of your last three exams were 75%, 81%, and 93%. What score did you receive on the first exam?

(20 pts) 5. __87%__

6. There are 39 butterflies in one section of a flower garden. This represents 6% of the population of all butterflies in the garden. How many butterflies are there in the entire garden?

(20 pts) 6. __650__

7. A bacterial culture doubles every 20 minutes. There are 22,528 bacteria at noon. How many bacteria were there at 10 am earlier that same day?

(20 pts) 7. __352__

8. What is the greatest common divisor of 360 and 525?

(20 pts) 8. __15__

9. What is the difference between the least positive odd number divisible by three distinct primes and the least positive even number divisible by three distinct primes?

(20 pts) 9. __75__

10. Suppose the price of an item is decreased by 30% and then later the new price is increased by 50%. Compared to the original price, what percentage increase does the final price represent?

(20 pts) 10. __5%__

School _____   **Key**   Team Name _____

Calculators are NOT allowed.

1. How many of the first 1000 positive integers are multiples of both 4 and 5 but not of 6?

(20 pts) 1. ___34___

2. If a radius of a circle whose area is 36π cm^2 equals the width of a rectangle, and the diameter of the circle is half the length of the rectangle, then what is the perimeter of the rectangle (in cm)?

(20 pts) 2. ___60___

3. A coin jar has 100 pennies, 200 nickels, 300 dimes, and 400 quarters in it. What is the total value (in dollars) of all the coins in the jar?

(20 pts) 3. ___141___

4. A box contains igneous, metamorphic and sedimentary rocks. $\dfrac{1}{3}$ of the rocks are igneous, 60 are metamorphic, and 40% are sedimentary. How many total rocks are in the box?

(20 pts) 4. ___225___

5. If I start with \$100 in my bank account and then increase my money by 20% in the first year, again increase my money by 20% in the second year, and then lose 50% of my money in the third year, how much money (in dollars) do I have left?

(20 pts) 5. ___72___

6. Evaluate
$$\left(\frac{1}{5} + \frac{5}{1} + \frac{5}{20} + \frac{20}{5}\right) - \left(\frac{1}{4} + \frac{4}{1} + \frac{2}{10} + \frac{10}{2}\right)$$

(20 pts) 6. ___0___

7. Sam cleared 12 levels on his computer game in 30 minutes. If she continues at this rate, how many minutes will it take her to clear 40 levels?

(20 pts) 7. ___100___

8. Consider the sequence of numbers 11, 28, 45 . . . , in which each number is 17 more than its predecessor. What is the 100th number in the sequence?

(20 pts) 8. ___1694___

9. If $-400 \le x \le -300$ and $400 \le y \le 1200$. What is the largest value of $\dfrac{y}{x}$?

(20 pts) 9. ___-1___

10. A drawer has 2 blue socks and 4 red socks. Another drawer has 6 blue socks and 8 red socks. If you take out a sock from each drawer, what is the probability that you will get a matching pair? Input your answer as a fraction in simplest form.

(20 pts) 10. ___$\dfrac{11}{21}$___

School ——————————

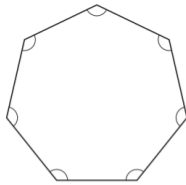Team Name ——————————

Calculators are allowed.

Student Name ——————————

1. What is the y coordinate of the y-intercept of the line perpendicular to the line $y = -\dfrac{1}{2}x + 5$ and passes through the point $(-4, 3)$?

(2 pts) 1. ————————

2. What is the sum, in degrees, of the measure of the interior angles of the polygon below?

(3 pts) 2. ————————

3. Ten years ago, the population of a town was 5,085 people. Since then the population of the town has increased by a total of 19.5% What is the current population of the town rounded to the nearest integer?

(3 pts) 3. ————————

4. The number of loaves of bread remaining in a restaurant h hours after opening for the day can be modeled by $L(h) = 42 - 3.5h$. What is the range of $L(h)$? Write your answer in the form $a \leq L(h) \leq b$.

(3 pts) 4. ————————

5. Given the function $f(x) = x^2 - x + 1$, find $f(m-1)$.

(3 pts) 5. ————————

6. Solve $|4 - 2x| \leq 6$. Write your answer in the form $a \leq x \leq b$.

(3 pts) 6. ————————

7. What is the vertex of the parabola $y = -\dfrac{1}{2}x^2 + 2x + 1$?

(3 pts) 7. ————————

14

jrs

TOTAL POINTS ————————

School _____   Team Name _____

Calculators are NOT allowed.   Student Name _____

1. What is the sum of two roots of the equation $1 + \dfrac{1}{x-1} = x + 1$?

   (2 pts) 1. _____

2. $x = 1$ is a root of the equation $x^3 + 2x^2 + x + k$ for some constant k. What is the product of all three roots of the equation?

   (3 pts) 2. _____

3. If $-2 \leq x \leq 3$ and $-4 \leq y \leq 5$, then $a \leq xy \leq b$. What is $a + b$?

   (3 pts) 3. _____

4. If a sequence $\{a_n\}$ of positive numbers converges to L and $a_{n+1}^2 - a_n = 2$, what is L?

   (3 pts) 4. _____

5. If $\big||x - 2| + 3\big| < 4$, then $a < x < b$. What is $a + b$?

   (3 pts) 5. _____

6. A box with a square base has height 5. If the surface area of the box is 78, what is the length of one side of the square?

   (3 pts) 6. _____

7. What is the probability that the sum of points will be 14 when three dice are rolled?
   (a) 4/71   (b) 5/72   (c) 7/74   (d) 8/75   (e) 9/76

   (3 pts) 7. _____

dh   TOTAL POINTS _____

School ——————   Team Name ——————

Calculators are allowed.   Student Name ——————

1. A line passes through the points A and B with coordinates $(1, 6)$ and $(1000, 36)$ respectively. How many other points with integer coordinates does the line pass through between A and B?

(2 pts) 1. ————

2. The number of real zeros of the polynomial equation, $f(x) = x^3 + x^2 + 3x + 3$ is?

(3 pts) 2. ————

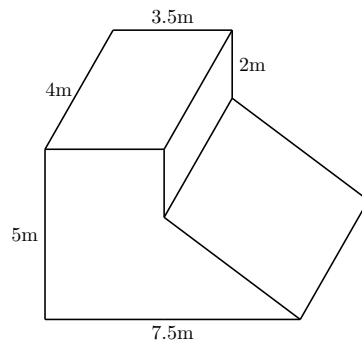3. For the figure below, how many rotations and reflections carry it onto itself?
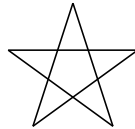
(3 pts) 3. ————

4. Solve the compound inequality below and express the solution in interval notation.

$$2x + 5 < 7 \quad \text{or} \quad 7x - 1 \geq -8$$

(3 pts) 4. ————

5. What is the surface area of the object below?

(3 pts) 5. ————

6. A student works as both a babysitter and a tutor and charges an hourly rate for each. One day the student earns a total of \$118 for 6 hours of babysitting and 2 hours of tutoring. On another day the student earns \$93 for 3 hours of babysitting and 3 hours of tutoring. How much does the student charge per hour as a babysitter?

(3 pts) 6. ————

7. Fifteen numbers have an average of 15. Five of those numbers have an average of 5, four others have an average of 4, three others have an average of 3 and two others have an average of 2. What is the remaining number?

(3 pts) 7. ————

mi   TOTAL POINTS ————

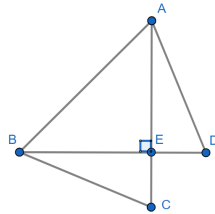School _____   Team Name _____

Calculators are NOT allowed.   Student Name _____

1. A number is divided by 6 and then 3 is subtracted from the result to give 4. What is the original number?

(2 pts) 1. _____

2. Triangles ABC and ABD are isosceles with $AB = AC = BD$, $\overline{BD}$ intersects $\overline{AC}$ at E and $\overline{BD} \perp \overline{AC}$. If possible, determine the value of $m\angle C + m\angle D$ (in degrees).

(3 pts) 2. _____

3. Determine the value of the expression $[2 - 3(2 - 3)^{-1}]^{-1}$.

(3 pts) 3. _____

4. A car travels one mile is one minute. To travel one mile in 40 seconds the car must increase its speed by ____ percent.

(3 pts) 4. _____

5. If $x + 2y = 5$, $z + 2x = 9$, and $y + 2z = 10$, find the value of $x + y + z$.

(3 pts) 5. _____

6. In a rectangle we decrease one side by 3 and increase the adjacent side by 2 to form a square of area 36. What is the perimeter of the rectangle?

(3 pts) 6. _____

7. A person's salary is decreased by 15%. By what percent would the reduced salary then have to be raised to bring it back to the original amount?

(3 pts) 7. _____

mei   TOTAL POINTS _____

School _____

Team Name _____

Calculators are allowed.

1. Factor the integer 2021 into primes.

   (20 pts) 1. _____

2. Determine the length, l of a rectangle, with width w, if its perimeter is 30, its area is 54, and $l > w$.

   (20 pts) 2. _____

3. Find the smallest integer n so that the remainder is 6 when 2^n is divided by 11.

   (20 pts) 3. _____

4. What is the remainder when $x^{56} + x^{48} + x^{26} + x^8 + 3$ is divided by $x^2 - 1$?

   (20 pts) 4. _____

5. A full house is a five-card poker hand consisting of three of one kind and two of another. What is the probability that a full house contains at least two kings?

   (20 pts) 5. _____

6. Find a solution in positive integers to $x^2 - 11y^2 = 1$.

   (20 pts) 6. _____

7. If $\log_8 y = 10$, what is $\sqrt{y}$?

   (20 pts) 7. _____

8. Given a whole number construct a sequence as follows. If the number is even, divide by 2; if it's odd multiply by three and add 1. Apply the same method to the result to get the next number. Stop if/when you reach 1. How many steps are required if you start with 18?

   (20 pts) 8. _____

9. Find a ten digit number which contains every digit 0,1,..., 9 exactly once, starts with a 3 and is divisible by every whole number between 2 and 18.

   (20 pts) 9. _____

10. If $x + y = 5, y + z = 6, z + w = 4$, and $x + 2w = 6$, find x.

    (20 pts) 10. _____

ji

TOTAL POINTS _____

School _____   Team Name _____

Calculators are NOT allowed.

1. Suppose you know $x + 3y = -1$ and $3y - x = -29$. What is xy?

(20 pts) 1. _____

2. Define $\#N$ by the formula $\#N = \dfrac{N}{3} - 2$. Calculate $\#(\#(\#51))$

(20 pts) 2. _____

3. The values of a, b, c, and d are 1, 2, 3, and 4, though not necessarily in that order. What is the greatest possible value of $ab + bc + cd + da$?

(20 pts) 3. _____

4. Four circles of radius 1 are drawn with centers at the points $(1, 0)$, $(-1, 0)$, $(0, 1)$, and $(0, -1)$. A circle centered at $(0, 0)$ and of radius 2 is drawn. What is the area of all points that are contained in an odd number of these 5 circles? Express your answer using π.

(20 pts) 4. _____

5. How many sets of 3 primes add up to 26?

(20 pts) 5. _____

6. What is the difference between the sum of the first 200 even natural numbers and the sum of the first 200 odd natural numbers?

(20 pts) 6. _____

7. There are 4! strings that can be formed using each of the letters 'a', 'b', 'c', and 'd' once. If these strings are alphabatized, which one appears 15th in the list?

(20 pts) 7. _____

8. The number n is randomly selected from the set $\{1, 2, \ldots, 10\}$, with each number being equally likely to be chosen. What is the probability that $3n - 1 > 2n + 6$?

(20 pts) 8. _____

9. The measures of the sides of a triangle are 7, 24, and 25. What is the area of the triangle?

(20 pts) 9. _____

10. A point is chosen at random on the coordinate plane such that both of its coordinates are between -1 and 1. What is the probability that the point is less than 1 unit from the origin? Express your answer using π.

(20 pts) 10. _____

rz   TOTAL POINTS _____

School _____   Team Name _____

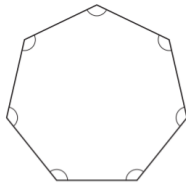Calculators are allowed.   **Key**   Student Name _____

1. What is the y coordinate of the y-intercept of the line perpendicular to the line $y = -\frac{1}{2}x + 5$ and passes through the point $(-4, 3)$?

(2 pts) 1. ____11____

2. What is the sum, in degrees, of the measure of the interior angles of the polygon below?

(3 pts) 2. ____900____

3. Ten years ago, the population of a town was 5,085 people. Since then the population of the town has increased by a total of 19.5% What is the current population of the town rounded to the nearest integer?

(3 pts) 3. ____6077____

4. The number of loaves of bread remaining in a restaurant h hours after opening for the day can be modeled by $L(h) = 42 - 3.5h$. What is the range of $L(h)$? Write your answer in the form $a \leq L(h) \leq b$.

(3 pts) 4. $\underline{0 \leq L(h) \leq 42}$

5. Given the function $f(x) = x^2 - x + 1$, find $f(m-1)$.

(3 pts) 5. $\underline{m^2 - 3m + 3}$

6. Solve $|4 - 2x| \leq 6$. Write your answer in the form $a \leq x \leq b$.

(3 pts) 6. $\underline{-1 \leq x \leq 5}$

7. What is the vertex of the parabola $y = -\frac{1}{2}x^2 + 2x + 1$?

(3 pts) 7. ____$(2, 3)$____

20

School _____   Team Name _____

Calculators are NOT allowed.   **Key**   Student Name _____

1. What is the sum of two roots of the equation $1 + \dfrac{1}{x-1} = x + 1$?

   (2 pts) 1. ____1____

2. $x = 1$ is a root of the equation $x^3 + 2x^2 + x + k$ for some constant k. What is the product of all three roots of the equation?

   (3 pts) 2. ____4____

3. If $-2 \le x \le 3$ and $-4 \le y \le 5$, then $a \le xy \le b$. What is $a + b$?

   (3 pts) 3. ____3____

4. If a sequence $\{a_n\}$ of positive numbers converges to L and $a_{n+1}^2 - a_n = 2$, what is L?

   (3 pts) 4. ____2____

5. If $\left| |x - 2| + 3 \right| < 4$, then $a < x < b$. What is $a + b$?

   (3 pts) 5. ____4____

6. A box with a square base has height 5. If the surface area of the box is 78, what is the length of one side of the square?

   (3 pts) 6. ____3____

7. What is the probability that the sum of points will be 14 when three dice are rolled?
   (a) 4/71   (b) 5/72   (c) 7/74   (d) 8/75   (e) 9/76

   (3 pts) 7. ____(b)____

School _____

Team Name _____

Calculators are allowed. **Key**
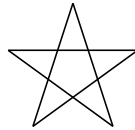
Student Name _____

1. A line passes through the points A and B with coordinates $(1, 6)$ and $(1000, 36)$ respectively. How many other points with integer coordinates does the line pass through between A and B?

   (2 pts) 1. ____2____

2. The number of real zeros of the polynomial equation, $f(x) = x^3 + x^2 + 3x + 3$ is?

   (3 pts) 2. ____1____

3. For the figure below, how many rotations and reflections carry it onto itself?

   (3 pts) 3. ____10____
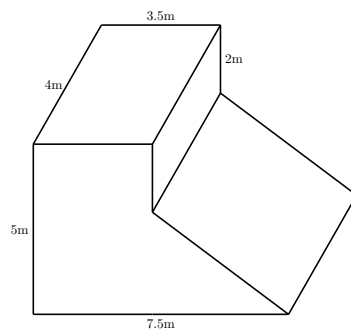
4. Solve the compound inequality below and express the solution in interval notation.
$$2x + 5 < 7 \quad \text{or} \quad 7x - 1 \geq -8$$

   (3 pts) 4. $(-\infty, \infty)$

5. What is the surface area of the object below?

   (3 pts) 5. 139 m^2

6. A student works as both a babysitter and a tutor and charges an hourly rate for each. One day the student earns a total of $118 for 6 hours of babysitting and 2 hours of tutoring. On another day the student earns $93 for 3 hours of babysitting and 3 hours of tutoring. How much does the student charge per hour as a babysitter?

   (3 pts) 6. ____$14____

7. Fifteen numbers have an average of 15. Five of those numbers have an average of 5, four others have an average of 4, three others have an average of 3 and two others have an average of 2. What is the remaining number?

   (3 pts) 7. ____171____

School ———————
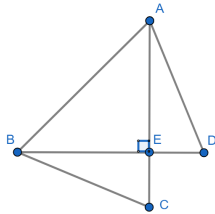
Team Name ———————

Calculators are NOT allowed.

Key

Student Name ———————

1. A number is divided by 6 and then 3 is subtracted from the result to give 4. What is the original number?

(2 pts) 1. ___42___

2. Triangles ABC and ABD are isosceles with $AB = AC = BD$, $\overline{BD}$ intersects $\overline{AC}$ at E and $\overline{BD} \perp \overline{AC}$. If possible, determine the value of $m\angle C + m\angle D$ (in degrees).

(3 pts) 2. ___135°___

3. Determine the value of the expression $[2 - 3(2 - 3)^{-1}]^{-1}$.

(3 pts) 3. ___$\dfrac{1}{5}$___

4. A car travels one mile is one minute. To travel one mile in 40 seconds the car must increase its speed by ____ percent.

(3 pts) 4. ___50%___

5. If $x + 2y = 5$, $z + 2x = 9$, and $y + 2z = 10$, find the value of $x + y + z$.

(3 pts) 5. ___8___

6. In a rectangle we decrease one side by 3 and increase the adjacent side by 2 to form a square of area 36. What is the perimeter of the rectangle?

(3 pts) 6. ___26___

7. A person's salary is decreased by 15%. By what percent would the reduced salary then have to be raised to bring it back to the original amount?

(3 pts) 7. ___$17\dfrac{11}{17}\%$___

School _____   **Key**   Team Name _____

Calculators are allowed.

1. Factor the integer 2021 into primes.

   (3 pts) 1. __$43 * 47$__

2. Determine the length, l of a rectangle, with width w, if its perimeter is 30, its area is 54, and $l > w$.

   (3 pts) 2. ____9____

3. Find the smallest integer n so that the remainder is 6 when 2^n is divided by 11.

   (3 pts) 3. ____9____

4. What is the remainder when $x^{56} + x^{48} + x^{26} + x^8 + 3$ is divided by $x^2 - 1$?

   (3 pts) 4. ____7____

5. A full house is a five-card poker hand consisting of three of one kind and two of another. What is the probability that a full house contains at least two kings?

   (3 pts) 5. $\dfrac{2}{13}$ or 15.38%

6. Find a solution in positive integers to $x^2 - 11y^2 = 1$.

   (3 pts) 6. $x = 10, y = 3$

7. If $\log_8 y = 10$, what is $\sqrt{y}$?

   (3 pts) 7. $8^5 = 32768$

8. Given a whole number construct a sequence as follows. If the number is even, divide by 2; if it's odd multiply by three and add 1. Apply the same method to the result to get the next number. Stop if/when you reach 1. How many steps are required if you start with 18?

   (3 pts) 8. ____20____

9. Find a ten digit number which contains every digit 0,1,..., 9 exactly once, starts with a 3 and is divisible by every whole number between 2 and 18.

   (3 pts) 9. 3785942160

10. If $x + y = 5, y + z = 6, z + w = 4$, and $x + 2w = 6$, find x.

   (3 pts) 10. ____0____

School _____   **Key**   Team Name _____

Calculators are NOT allowed.

1. Suppose you know $x + 3y = -1$ and $3y - x = -29$. What is xy?

   (20 pts) 1. ___-70___

2. Define $\#N$ by the formula $\#N = \dfrac{N}{3} - 2$. Calculate $\#(\#(\#51))$

   (20 pts) 2. ___-1___

3. The values of a, b, c, and d are 1, 2, 3, and 4, though not necessarily in that order. What is the greatest possible value of $ab + bc + cd + da$?

   (20 pts) 3. ___25___

4. Four circles of radius 1 are drawn with centers at the points $(1,0)$, $(-1,0)$, $(0,1)$, and $(0,-1)$. A circle centered at $(0,0)$ and of radius 2 is drawn. What is the area of all points that are contained in an odd number of these 5 circles? Express your answer using π.

   (20 pts) 4. ___$4\pi - 8$___

5. How many sets of 3 primes add up to 26?

   (20 pts) 5. ___3___

6. What is the difference between the sum of the first 200 even natural numbers and the sum of the first 200 odd natural numbers?

   (20 pts) 6. ___200___

7. There are 4! strings that can be formed using each of the letters 'a', 'b', 'c', and 'd' once. If these strings are alphabatized, which one appears 15th in the list?

   (20 pts) 7. ___cbad___

8. The number n is randomly selected from the set $\{1, 2, \ldots, 10\}$, with each number being equally likely to be chosen. What is the probability that $3n - 1 > 2n + 6$?

   (20 pts) 8. $\dfrac{3}{10}$ or 0.3

9. The measures of the sides of a triangle are 7, 24, and 25. What is the area of the triangle?

   (20 pts) 9. ___84___

10. A point is chosen at random on the coordinate plane such that both of its coordinates are between -1 and 1. What is the probability that the point is less than 1 unit from the origin? Express your answer using π.

    (20 pts) 10. $\dfrac{\pi}{4}$

School —————————

Team Name —————————

Calculators are allowed.

Student Name —————————

1. Let $p(x) = (x+1)(x+2)(x+3)\cdots(x+2022)$. What is the coefficient of x^{2021}?

   (2 pts) 1. —————

2. Peter had a 12:00 noon appointment that was 60 miles from his home. He drove from his home at an average of 40 miles per hour and arrived 15 minutes late. At what time did Peter leave home for the appointment?

   (3 pts) 2. —————

3. Suppose you want to make a paper cone by taking a $\dfrac{3}{4}$ of a paper circle of radius 1 inch and joining two radii. What is the height of the cone? Give your answer in inches rounded in two decimal places.

1 inch
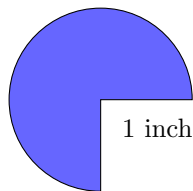
   (3 pts) 3. —————

4. A right triangle has perimeter 24 and area 24. What is the length of its hypotenuse?

   (3 pts) 4. —————

5. The set of the odd consecutive numbers 1, 3, 5, 7, $\cdots$, N has a sum of 400. How many numbers are in the set?

   (3 pts) 5. —————

6. Albert and Gauss found the solution separately for the following system of two equations:

$$\begin{cases} Ax - y = 7 \\ 2x + By = 9 \end{cases}$$

   Albert got the solution $(x, y) = (-30, 23)$ with the wrong value for A used by accident. On the other hand, Gauss got the solution $(x, y) = (12, 5)$ with the wrong value for B used. What is the correct solution (x, y) of the system?

   (3 pts) 6. —————

7. Find the sum of the first 100 terms of the sequence $\{a_n\}$ defined by

$$a_{n+1} = \begin{cases} 2a_n, & a_n < 7 \\ a_n - 7, & a_n \geq 7 \end{cases}$$

   and the first term $a_1 = 1$.

   (3 pts) 7. —————

26

ejy

TOTAL POINTS —————

School _____   Team Name _____

Calculators are NOT allowed.   Student Name _____

1. Two circles C_1, C_2 have radii 6 cm and 8 cm, respectively. A third circle's area is the same as the sum of the areas of C_1 and C_2. The radius of the third circle in cm is:

   (a) 10   (b) 11   (c) 12   (d) 13   (e) 14   (2 pts) 1. _____

2. How many integers satisfy the inequality $\dfrac{x^2 - 61}{x^2 - 16} \leq 0$?

   (a) 3   (b) 4   (c) 6   (d) 8   (e) infinitely many   (3 pts) 2. _____

3. Two rectangles have the same area. In the first rectangle the ratio of the long edge to the short edge is 3 : 1 and in the second rectangle these edges are in a ratio of 4 : 3. Then the ratio of the perimeter of the first rectangle to the perimeter of the second rectangle is:

   (a) 2 : 3   (b) 8 : 7   (c) 7 : 4   (d) 13 : 6   (e) not uniquely determined   (3 pts) 3. _____

4. Let f be the function such that $f(x) = ax^9 + bx^5 + cx^3 - 7$, where a, b, and c are real numbers. If $f(-9) = 9$ then $f(9)$ is equal to

   (a) -23 (b) -9 (c) -5 (d) 23   (e) insufficient information   (3 pts) 4. _____

5. A doctor's office has a row of chairs, two of which are already occupied. Ruby and Ollie come in and want to sit somewhere in the row, in adjacent chairs, that is, beside each other without a gap. Because of social distancing, there must be at least two empty chairs between them and the persons already sitting. How many chairs at least must the row contain so that Ruby and Ollie can always find a place to sit together, regardless of the location of the occupied chairs?

   (a) 10   (b) 11   (c) 12   (d) 13   (e) 14   (3 pts) 5. _____

6. For how many integers $1 \leq n \leq 100$ is n^n a perfect square?

   (a) 10   (b) 50   (c) 55   (d) 60   (e) none of these   (3 pts) 6. _____

7. Jane has 200 marbles of various colors. She splits them into groups of the same color, and she observes that each group has a different number of marbles. The maximum number of groups she can have is

   (a) 19   (b) 21   (c) 23   (d) 24   (e) none of these   (3 pts) 7. _____

   bgd   TOTAL POINTS _____

School _____ Team Name _____

Calculators are allowed. Student Name _____
All answers should be exact or rounded to three significant figures.

1. Celsius and Fahrenheit temperatures are related by $F = \frac{9}{5}C + 32$. What is the (positive)
   Fahrenheit temperature that is the negation of its equivalent Celsius temperature? (2pts) 1. _____

2. What is the probability that if you flip a coin six times,
   there are at least as many heads as tails? (3pts) 2. _____

3. Find the zeroes of the expression
   $(1 - x)(5 - x)(9 - x) + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8$
   $\quad - (1 - x) \cdot 6 \cdot 8 - (9 - x) \cdot 2 \cdot 4 - (5 - x) \cdot 3 \cdot 7$ (3pts) 3. _____

4. How many positive integers evenly divide 2022? (3pts) 4. _____

5. Find x and y, given that
$$x + y = 2$$
$$x^2 + xy + y^2 = 19$$
$$x > y$$
   (3pts) 5. _____

6. Size A0 paper can be defined as being $10\,000\,\text{cm}^2$ in area and having dimensions
   so that cutting the paper in half gives two sheets that are similar to the original.
   What are the length and width of A0 paper in centimeters? (3pts) 6. _____

7. What is the smallest n so that the integers $1, 2, \ldots, n$ can be placed in the
   circles below so that no line connects two circles with the same number?

   (3pts) 7. _____

TOTAL POINTS _____
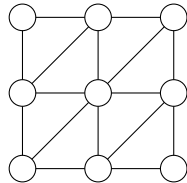
School _____

Team Name _____

Calculators are NOT allowed.

Student Name _____

1. Suppose N is a number such that $N = 24_b$ and $N^2 = 554_b$. What is the base b? (Assume $b > 0$)

(2 pts) 1. _____

2. A digital 12-hr clock shows the time 4:56. How many minutes will pass until the clock next shows a time in which all of the digits are consecutive and are in increasing order?

(3 pts) 2. _____

3. A point is located in the interior of a rectangle so that its distances from two opposite corners are 5 units and 14 units, while its distance from a third corner is 10 units. What is its distance from the fourth corner?

(3 pts) 3. _____

4. A fair die is rolled 6 times. Let p be the probability that each of the six faces on the die appears exactly once among the six rolls. Which of the following is correct?

   (a) $p \leq 0.02$
   (b) $0.02 < p \leq 0.04$
   (c) $0.04 < p \leq 0.06$
   (d) $0.06 < p \leq 0.08$
   (e) $p > 0.08$

(3 pts) 4. _____

5. A linear function of the form $f(x) = ax + b$ (where $a \neq 0$) satisfies the equation $f(x) - f^{-1}(x) = 44$ for all x. Find the value of $a + b$.

(3 pts) 5. _____

6. A ticket fee was $10, but then it was reduced. The number of customers increased by 50%, but the amount of money received only increased by 20%. How many dollars was the reduced ticket price?

(3 pts) 6. _____

7. What is the maximum value of the function $f(x) = \dfrac{\sin^3 x \cos x}{\tan^2 x + 1}$?

(3 pts) 7. _____

rp

TOTAL POINTS _____
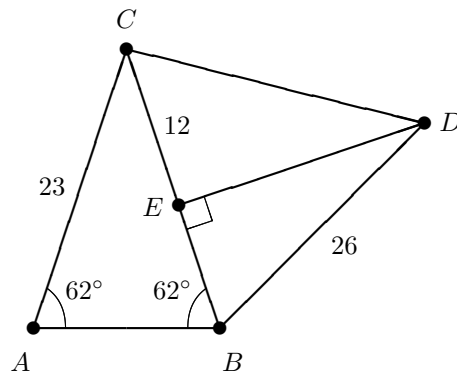
School _____   Team Name _____

Calculators are allowed.

1. A rectangular garden is 31 feet long and 17 feet wide. Find the area of the garden. Express your answer in square feet.

(20 pts) 1. _____

2. In the above figure, suppose that the distance from A to C is 23, the distance from E to C is 12, and the distance from B to D is 26. Also suppose that $\angle CAB$ and $\angle CBA$ have measure $62°$ and $\angle BED$ has measure $90°$. Find the area of ΔBCD. Round your answer to one decimal place.

(20 pts) 2. _____

3. A kettle is filled with hot water. The temperature of the water in the kettle t minutes from now will be

$$T(t) = \frac{380}{5 - 3e^{-t/4}} \quad \text{degrees Fahrenheit.}$$

After how many minutes will the temperature of the water be 82.0 degrees Fahrenheit? Round your answer to the nearest tenth of a minute.

(20 pts) 3. _____

4. A large jar contains 25 balls. Of these 25 balls, 12 are red, 8 are yellow, and 5 are blue. Suppose that Mrs. Hemmami plans to remove two balls from the jar, leaving 23 balls in the jar. What is the probability that the two balls will be of different colors? Express your answer as a decimal number, and round your answer to two decimal places.

(20 pts) 4. _____

5. A movie theater charges $12 for a ticket to a movie. Senior citizens receive a discount, however. The price of one movie ticket for a senior citizen is only $7. On a certain day, the theater sold 54 tickets, and the total ticket sales revenue for the day was $603. How many tickets for senior citizens were sold?

(20 pts) 5. _____

6. A password consists of five characters. Two of the five characters must be capital letters, and the other three characters must be digits (such as 1, 2, 3, and so forth). The capital letters must not appear in adjacent positions within the password. For example, E46J2 is a valid password, but 46EJ2 is *not* a valid password. How many different passwords are possible?

   Hint: Our language has 26 letters and our number system has ten digits.

   (20 pts) 6. _____

7. Mrs. Englekey is designing a box. The dimensions of the box are to be x inches by $8 - 2x$ inches by $12 - 2x$ inches. Here x is a number which Mrs. Englekey has not yet determined. Find the value of x which will result in the box with the largest possible volume. Round your answer to two decimal places.

   Hint: All three dimensions must be positive real numbers.

   (20 pts) 7. _____

8. Let constant real numbers A, B, C, and D be given, and suppose that $A \neq 0$. For all real numbers x, let

$$P(x) = Ax^3 + Bx^2 + Cx + D$$

   Suppose that $P(k) = 2^k$ for $k = 0, 1, 2,$ and 3. Determine $P(4)$.

   (20 pts) 8. _____

9. Suppose that x and y are positive real numbers. Suppose also that $x - 2y > 0$ and $2\ln(x - 2y) = \ln x + \ln y$. Find $\frac{x}{y}$.

   (20 pts) 9. _____

10. A train leaves a station precisely on the minute. After the train has traveled exactly 8 miles, the driver consults her watch and sees that the hour-hand is directly over the minute-hand. The average speed over the 8 miles was 33 miles per hour. At what time did the train leave the station?

   Note 1: You should assume that the driver's watch is an old-fashioned watch. It has an hour-hand, a minute-hand, and a second-hand. Note, for example, that at 12:00 o'clock, all three hands point to 12. As another example, note that at exactly the time 3:05, the second-hand points to 12, the minute-hand points to 5, and the hour-hand points slightly past 3.

   Note 2: The problem says that the train left the station precisely on the minute. This means that at the moment that the train left the station, the second-hand pointed to 12.

   Note 3: Do not worry about whether it is morning or afternoon or what day it is. This problem only concerns the times indicated on the driver's watch.

   (20 pts) 10. _____

lp   TOTAL POINTS _____

School _____   Team Name _____

Calculators are NOT allowed.

1. What number is one third of the way from $\frac{1}{4}$ to $\frac{3}{4}$?

(20 pts) 1. _____

2. How many positive divisors does 1 million have? (Count them, not list them)

(20 pts) 2. _____

3. It takes Anna 25 minutes to walk 1.5 mile from home to school, but only 15 minutes to walk from school to home along the same route. What is Anna's average speed, in miles per hour, for the round trip?

(20 pts) 3. _____

4. The sum of the two 5-digit numbers $UND20$ and $UND22$ is 170442. What is the sum $U + N + D$?

(20 pts) 4. _____

5. The graph of the polynomial $p(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$ has five distinct x-intercepts, one of which is at $(0,0)$. Which one of the coefficients (a, b ,c, d, e) cannot be zero?

(20 pts) 5. _____

6. Suppose a square has perimeter P. Let A be the area of a circle circumscribed about the square. Find an expression for A in terms of P.

(20 pts) 6. _____

7. A certain donut shop only sells donuts in combinations of 4-packs and 9-packs. The shop can fill an order of 16 donuts as four 4-packs, but not an order of 15 donuts since 15 cannot be expresses as nonnegative multiples of 4-packs and 9-packs. What is the largest number of donuts ordered which cannot be filled by the shop?

(20 pts) 7. _____

8. The first three terms of an arithmetic sequence are $2y - 3$, $5y - 11$, and $3y + 1$ respectively. The nth term of the sequence is 2021. What is n?

(20 pts) 8. _____

9. One dimension of a cube is increased by 1, another is decreased by 1, and the third is left unchanged. The volume of the new rectangular solid is 4 less than that of the cube. What was the volume of the cube?

(20 pts) 9. _____

10. Recall $i = \sqrt{-1}$. Find the positive integer n so that

$$i + 2i^2 + 3i^3 + \cdots + ni^n = 20 + 21i.$$

(20 pts) 10. _____

jb   TOTAL POINTS _____

School _____

Team Name _____

Calculators are allowed. **Key**

Student Name _____

1. Let $p(x) = (x+1)(x+2)(x+3)\cdots(x+2022)$. What is the coefficient of x^{2021}?

   (2 pts) 1. 2,045,253

2. Peter had a 12:00 noon appointment that was 60 miles from his home. He drove from his home at an average of 40 miles per hour and arrived 15 minutes late. At what time did Peter leave home for the appointment?

   (3 pts) 2. 10:45 a.m

3. Suppose you want to make a paper cone by taking a $\dfrac{3}{4}$ of a paper circle of radius 1 inch and joining two radii. What is the height of the cone? Give your answer in inches rounded in two decimal places.

   1 inch
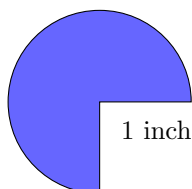
   (3 pts) 3. $\dfrac{\sqrt{7}}{4} \approx 0.66$ inches

4. A right triangle has perimeter 24 and area 24. What is the length of its hypotenuse?

   (3 pts) 4. 10

5. The set of the odd consecutive numbers 1, 3, 5, 7, $\cdots$, N has a sum of 400. How many numbers are in the set?

   (3 pts) 5. 20

6. Albert and Gauss found the solution separately for the following system of two equations:

$$\begin{cases} Ax - y &= 7 \\ 2x + By &= 9 \end{cases}$$

   Albert got the solution $(x, y) = (-30, 23)$ with the wrong value for A used by accident. On the other hand, Gauss got the solution $(x, y) = (12, 5)$ with the wrong value for B used. What is the correct solution (x, y) of the system?

   (3 pts) 6. $(x, y) = (6, -1)$

7. Find the sum of the first 100 terms of the sequence $\{a_n\}$ defined by

$$a_{n+1} = \begin{cases} 2a_n, & a_n < 7 \\ a_n - 7, & a_n \geq 7 \end{cases}$$

   and the first term $a_1 = 1$.

   33

   (3 pts) 7. 375

School _____

Team Name _____

Calculators are NOT allowed.   **Key**

Student Name _____

1. Two circles C_1, C_2 have radii 6 cm and 8 cm, respectively. A third circle's area is the same as the sum of the areas of C_1 and C_2. The radius of the third circle in cm is:

   (a) 10   (b) 11   (c) 12   (d) 13   (e) 14   (2 pts) 1. ___A___

2. How many integers satisfy the inequality $\dfrac{x^2 - 61}{x^2 - 16} \leq 0$?

   (a) 3   (b) 4   (c) 6   (d) 8   (e) infinitely many   (3 pts) 2. ___C___

3. Two rectangles have the same area. In the first rectangle the ratio of the long edge to the short edge is 3 : 1 and in the second rectangle these edges are in a ratio of 4 : 3. Then the ratio of the perimeter of the first rectangle to the perimeter of the second rectangle is:

   (a) 2 : 3   (b) 8 : 7   (c) 7 : 4   (d) 13 : 6   (e) not uniquely determined   (3 pts) 3. ___B___

4. Let f be the function such that $f(x) = ax^9 + bx^5 + cx^3 - 7$, where a, b, and c are real numbers. If $f(-9) = 9$ then $f(9)$ is equal to

   (a) -23 (b) -9 (c) -5 (d) 23   (e) insufficient information   (3 pts) 4. ___A___

5. A doctor's office has a row of chairs, two of which are already occupied. Ruby and Ollie come in and want to sit somewhere in the row, in adjacent chairs, that is, beside each other without a gap. Because of social distancing, there must be at least two empty chairs between them and the persons already sitting. How many chairs at least must the row contain so that Ruby and Ollie can always find a place to sit together, regardless of the location of the occupied chairs?

   (a) 10   (b) 11   (c) 12   (d) 13   (e) 14   (3 pts) 5. ___E___

6. For how many integers $1 \leq n \leq 100$ is n^n a perfect square?

   (a) 10   (b) 50   (c) 55   (d) 60   (e) none of these   (3 pts) 6. ___C___

7. Jane has 200 marbles of various colors. She splits them into groups of the same color, and she observes that each group has a different number of marbles. The maximum number of groups she can have is

   (a) 19   (b) 21   (c) 23   (d) 24   (e) none of these   (3 pts) 7. ___A___

SOLUTION KEY   **SOLUTION KEY**   **SOLUTION KEY**

School _____   Team Name _____

Calculators are allowed.   Student Name _____
All answers should be exact or rounded to three significant figures.

1. Celsius and Fahrenheit temperatures are related by $F = \frac{9}{5}C + 32$. What is the (positive)
   Fahrenheit temperature that is the negation of its equivalent Celsius temperature?   (2pts) 1. _____ $80/7\,°\mathrm{F} = 11 + \frac{3}{7}\,°\mathrm{F} \approx 11.4\,°\mathrm{F}$

2. What is the probability that if you flip a coin six times,
   there are at least as many heads as tails?   (3pts) 2. _____ $\frac{42}{64} = \frac{21}{32} = 0.65625 \approx 0.656$
   Hint: $\left(\binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}\right)/2^6$

3. Find the zeroes of the expression
   $(1-x)(5-x)(9-x) + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8$
   $\quad - (1-x) \cdot 6 \cdot 8 - (9-x) \cdot 2 \cdot 4 - (5-x) \cdot 3 \cdot 7$ (3pts) 3. _____ $0, \frac{15 \pm 3\sqrt{33}}{2} \approx 16.1, -1.17$
   Hint: $= 18x + 15x^2 - x^3 = -x(x^2 - 15x - 18)$

4. How many positive integers evenly divide 2022?   (3pts) 4. _____   8
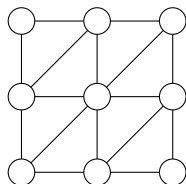   *Hint:* $2022 = 2 \cdot 3 \cdot 337$, *so the factors are* $2^i 3^j 337^k$, *where* $i, j, k \in \{0, 1\}$

5. Find x and y, given that
$$x + y = 2$$
$$x^2 + xy + y^2 = 19$$
$$x > y$$
          (3pts) 5. _____ $x = 5, y = -3.$

6. Size A0 paper can be defined as being $10\,000\,\mathrm{cm}^2$ in area and having dimensions
   so that cutting the paper in half gives two sheets that are similar to the original.
   What are the length and width of A0 paper in centimeters?   (3pts) 6. _____ $100 \cdot 2^{1/4}\mathrm{cm} \times 100/2^{1/4}\mathrm{cm} \approx 119\,\mathrm{cm} \times 84.1\,\mathrm{cm}$
   Hint: $L/W = 2W/L$, $WL = 10^4$

7. What is the smallest n so that the integers $1, 2, \ldots, n$ can be placed in the
   circles below so that no line connects two circles with the same number?

          (3pts) 7. _____   3

SOLUTION KEY   **SOLUTION KEY**   **SOLUTION KEY**
tp   TOTAL POINTS _____

School _____

Team Name _____

Calculators are NOT allowed.   **Key**   Student Name _____

1. Suppose N is a number such that $N = 24_b$ and $N^2 = 554_b$. What is the base b?
   (Assume $b > 0$)

   (2 pts) 1. ___12___

2. A digital 12-hr clock shows the time 4:56. How many minutes will pass until the clock next shows a time in which all of the digits are consecutive and are in increasing order?

   (3 pts) 2. ___458___

3. A point is located in the interior of a rectangle so that its distances from two opposite corners are 5 units and 14 units, while its distance from a third corner is 10 units. What is its distance from the fourth corner?

   (3 pts) 3. ___11___

4. A fair die is rolled 6 times. Let p be the probability that each of the six faces on the die appears exactly once among the six rolls. Which of the following is correct?

   (a) $p \leq 0.02$
   (b) $0.02 < p \leq 0.04$
   (c) $0.04 < p \leq 0.06$
   (d) $0.06 < p \leq 0.08$
   (e) $p > 0.08$

   (3 pts) 4. (a) or $p \leq 0.02$

5. A linear function of the form $f(x) = ax + b$ (where $a \neq 0$) satisfies the equation $f(x) - f^{-1}(x) = 44$ for all x. Find the value of $a + b$.

   (3 pts) 5. ___23___

6. A ticket fee was \$10, but then it was reduced. The number of customers increased by 50%, but the amount of money received only increased by 20%. How many dollars was the reduced ticket price?

   (3 pts) 6. ___8___

7. What is the maximum value of the function $f(x) = \dfrac{\sin^3 x \cos x}{\tan^2 x + 1}$?

   (3 pts) 7. 0.125 or $\dfrac{1}{8}$
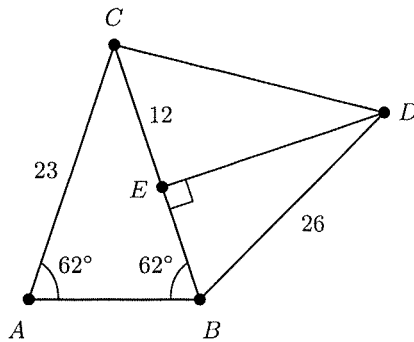
(handwritten: Trey)

School _____

Team Name _____

Calculators are allowed.

(handwritten: 527 ft.² (20 points)
527   (15 points)

1. A rectangular garden is 31 feet long and 17 feet wide. Find the area of the garden. Express your answer in square feet.

(20 pts) 1. _____

2. In the above figure, suppose that the distance from A to C is 23, the distance from E to C is 12, and the distance from B to D is 26. Also suppose that $\angle CAB$ and $\angle CBA$ have measure 62° and $\angle BED$ has measure 90°. Find the area of $\triangle BCD$. Round your answer to one decimal place.

(20 pts) 2. *270.9*

3. A kettle is filled with hot water. The temperature of the water in the kettle t minutes from now will be

$$T(t) = \frac{380}{5 - 3e^{-t/4}} \text{ degrees Fahrenheit.}$$

After how many minutes will the temperature of the water be 82.0 degrees Fahrenheit? Round your answer to the nearest tenth of a minute.

(handwritten: 8.4   (20 points)
8.4 minutes
(20 points)

(20 pts) 3. _____

4. A large jar contains 25 balls. Of these 25 balls, 12 are red, 8 are yellow, and 5 are blue. Suppose that Mrs. Hemmami plans to remove two balls from the jar, leaving 23 balls in the jar. What is the probability that the two balls will be of different colors? Express your answer as a decimal number, and round your answer to two decimal places.

(20 pts) 4. *0.65*

5. A movie theater charges $12 for a ticket to a movie. Senior citizens receive a discount, however. The price of one movie ticket for a senior citizen is only $7. On a certain day, the theater sold 54 tickets, and the total ticket sales revenue for the day was $603. How many tickets for senior citizens were sold?

(handwritten: 9 (20 points)
9 tickets

(20 pts) 5. _____ *(20 points*

6. A password consists of five characters. Two of the five characters must be capital letters, and the other three characters must be digits (such as 1, 2, 3, and so forth). The capital letters must not appear in adjacent positions within the password. For example, E46J2 is a valid password, but 46EJ2 is *not* a valid password. How many different passwords are possible?

Hint: Our language has 26 letters and our number system has ten digits.

4,056,000 (20 points)

4,056,000 passwords

(20 pts) 6. _____ (20 points)

7. Mrs. Englekey is designing a box. The dimensions of the box are to be x inches by $8 - 2x$ inches by $12 - 2x$ inches. Here x is a number which Mrs. Englekey has not yet determined. Find the value of x which will result in the box with the largest possible volume. Round your answer to two decimal places.

Hint: All three dimensions must be positive real numbers.

(20 pts) 7. 1.57

8. Let constant real numbers A, B, C, and D be given, and suppose that $A \neq 0$. For all real numbers x, let

$$P(x) = Ax^3 + Bx^2 + Cx + D$$

Suppose that $P(k) = 2^k$ for $k = 0$, 1, 2, and 3. Determine $P(4)$.

(20 pts) 8. 15

9. Suppose that x and y are positive real numbers. Suppose also that $x - 2y > 0$ and $2\ln(x - 2y) = \ln x + \ln y$. Find $\frac{x}{y}$.

(20 pts) 9. 4

10. A train leaves a station precisely on the minute. After the train has traveled exactly 8 miles, the driver consults her watch and sees that the hour-hand is directly over the minute-hand. The average speed over the 8 miles was 33 miles per hour. At what time did the train leave the station?

Note 1: You should assume that the driver's watch is an old-fashioned watch. It has an hour-hand, a minute-hand, and a second-hand. Note, for example, that at 12:00 o'clock, all three hands point to 12. As another example, note that at exactly the time 3:05, the second-hand points to 12, the minute-hand points to 5, and the hour-hand points slightly past 3.

Note 2: The problem says that the train left the station precisely on the minute. This means that at the moment that the train left the station, the second-hand pointed to 12.

Note 3: Do not worry about whether it is morning or afternoon or what day it is. This problem only concerns the times indicated on the driver's watch.

10:40 (20 points)

40 minutes after 10 o'clock

(20 points)

(20 pts) 10. _____

TOTAL POINTS _____

School _____ **Key** Team Name _____

Calculators are NOT allowed.

1. What number is one third of the way from $\frac{1}{4}$ to $\frac{3}{4}$?

(20 pts) 1. ___$\frac{5}{12}$___

2. How many positive divisors does 1 million have? (Count them, not list them)

(20 pts) 2. ___49___

3. It takes Anna 25 minutes to walk 1.5 mile from home to school, but only 15 minutes to walk from school to home along the same route. What is Anna's average speed, in miles per hour, for the round trip?

(20 pts) 3. _4.5 miles per hour_

4. The sum of the two 5-digit numbers $UND20$ and $UND22$ is 170442. What is the sum $U + N + D$?

(20 pts) 4. ___15___

5. The graph of the polynomial $p(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$ has five distinct x-intercepts, one of which is at $(0,0)$. Which one of the coefficients (a, b, c, d, e) cannot be zero?

(20 pts) 5. ___d___

6. Suppose a square has perimeter P. Let A be the area of a circle circumscribed about the square. Find an expression for A in terms of P.

(20 pts) 6. $A = $ ___$\frac{\pi P^2}{32}$___ or $\frac{\pi P^2}{32}$

7. A certain donut shop only sells donuts in combinations of 4-packs and 9-packs. The shop can fill an order of 16 donuts as four 4-packs, but not an order of 15 donuts since 15 cannot be expresses as nonnegative multiples of 4-packs and 9-packs. What is the largest number of donuts ordered which cannot be filled by the shop?

(20 pts) 7. ___23___

8. The first three terms of an arithmetic sequence are $2y - 3$, $5y - 11$, and $3y + 1$ respectively. The nth term of the sequence is 2021. What is n?

(20 pts) 8. ___505___

9. One dimension of a cube is increased by 1, another is decreased by 1, and the third is left unchanged. The volume of the new rectangular solid is 4 less than that of the cube. What was the volume of the cube?

(20 pts) 9. ___64___

10. Recall $i = \sqrt{-1}$. Find the positive integer n so that

$$i + 2i^2 + 3i^3 + \cdots + ni^n = 20 + 21i.$$

(20 pts) 10. ___41___