2026 Math Track Meet

University of North Dakota

January 12, 2026

Contents

Grades 7 & 8 Tests

UND MATHEMATICS TRACK MEET — INDIVIDUAL TEST 1
University of North Dakota — Grades 7/8
January 12, 2026

School ______________ Team Name ______________

Calculators are allowed. Student Name ______________

1. Simplify $(8-5)\times[(7+4)-(6\div 3)]$. Your answer should be an integer or a fraction where the numerator and denominator have no common factors.

   (2 pts) 1. ________

2. Each student in a class has a dog, a cat, or both a dog and a cat. Sixteen students have a dog (with or without a cat), and 10 have a cat (with or without a dog). If there are 21 students in all, how many have both a dog and a cat?

   (3 pts) 2. ________

3. The 21 students in a morning math class have an average score of 86 on a test. The 18 students in the afternoon class have an average score of 73. What is the average score of all 39 students together?

   (3 pts) 3. ________

4. A rectangular prism is 8 cm long, 5 cm wide, and 4 cm high. What is the total surface area of all faces of the prism?

   (3 pts) 4. ________

5. A line passes through three points (1, 2), (7, 4), and (16, a). What is the value of a?

   (3 pts) 5. ________

6. A box contains 3 red balls and 3 blue balls. Alice randomly draws and keeps one of the balls from the box, and then Bill randomly draws a different ball from those left in the box. What is the probability that Alice and Bill both draw red balls? Express your answer as a fraction where the numerator and denominator have no common factors.

   (3 pts) 6. ________

7. A square has sides of length 5 inches. A circle has a diameter of 6 inches. How much longer (in inches) is the perimeter of the square than the circumference of the circle? Use 3.14 for pi, and write your answer to the hundredth place.

   (3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 2
Grades 7/8

School _______________

Team Name _______________

Calculators are **NOT** allowed.

Student Name _______________

1. Simplify the expression
$$\left(\frac{2}{3}\right)^2 - \left(-\frac{3}{5}\right)^2 \div \frac{27}{25}$$

(2 pts) 1. _________

2. Simplify the expression
$$\sqrt{24} - 7\sqrt{54} + 6\sqrt{6}$$

(3 pts) 2. _________

3. Factor completely over reals
$$16x^5 - 81x$$

(3 pts) 3. _________

4. Solve for x and write your answer in set notation, i.e. { }.
$$x - 4x^{1/2} = 0$$

(3 pts) 4. _________

5. Solve the inequality $3 < 2x + 2 \leq 6$.

(3 pts) 5. _________

6. A die is tossed. What is the probability of tossing a 2 or a 5?

(3 pts) 6. _________

7. In 1990, Mary had a rectangular garden that measured a width of W ft and length of 20 ft. In the year 2000, she added 2 ft to the width of her flower garden. In 2010, she added another 3 ft to the width of her garden. What is the change in area from 1990 to 2010?

(3 pts) 7. _________

TOTAL POINTS _________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 3
Grades 7/8

School ______________

Team Name ______________

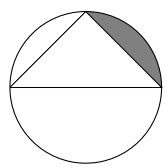
Calculators are allowed.

Student Name ______________

1. Two fair 6-sided dice are rolled and their product is recorded. Find the probability that their product is a multiple of 3. Round your answer to two decimal places.

(2 pts) 1. ________

2. An isosceles right triangle is inscribed in a circle as shown below. Given that the area of the circle is 9π, find the area of the shaded region. Round your answer to two decimal places.



(3 pts) 2. ________

3. Suppose that the population of a bacterial colony is currently 1248. If the population shrinks to half of its size every hour, how many hours will it be until the population reaches 156?

(3 pts) 3. ________

4. Quinn invested \$500 in a fund which is growing at a rate of 3.5% per month and \$1000 in a fund which is shrinking at a rate of 2% per month. By how much did Quinn's investment change after one month?

(3 pts) 4. ________

5. Find the set of all x for which $(2^x - 3)^2 + (2^x - 3) = 2$.

(3 pts) 5. ________

6. Suppose the date of the third Friday of October is two times the date of the second Tuesday of October. What day of the week is October 5th?

(3 pts) 6. ________

7. Starting with the number 5/7, we will begin repeating the following process. If the number is bigger than 1, subtract 1. Otherwise we multiply it by 2. Then repeat this again with the new number. How many repetitions do we make before we arrive back at 5/7?

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 4
Grades 7/8

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Student Name ______________

1. Evaluate $(-100)^0 + 3\mid -4^2 - (-8)\mid +2\mid \sqrt{16} + (-6)^2 \mid$

(2 pts) 1. ________

2. A chess club has only ninth-grade and tenth-grade students. If $\frac{3}{7}$ of the students are in ninth grade and there are 18 more tenth-grade students than ninth-grade students, how many tenth-grade students are in the club?

(3 pts) 2. ________

3. Two fair six-sided dice are rolled. What is the probability that the sum of the two numbers is at most 5?

(3 pts) 3. ________

4. A rectangular storage container is 30 feet long, 15 feet wide, and 10 feet deep. How many gallons of paint are needed to paint the outside of the four walls and the bottom if one gallon covers 150 square feet?

(3 pts) 4. ________

5. A bakery uses 4 mixing machines. Each machine mixes 18 batches of dough all together every $\frac{3}{4}$ minute. How many batches can all 4 machines mix in 2 minutes?

(3 pts) 5. ________

6. Find the x-intercept that is less than 0, if any, for

$$f(x) = (x-3)^2 - 9$$

(3 pts) 6. ________

7. Simplify the expression. Write your answer as a fraction in simplified form.

$$\frac{5^3 \cdot 25^2}{125^3} \times \left(\frac{4^2 \cdot 64}{8^4}\right)^{\frac{1}{2}}$$

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

TEAM TEST 1
Grades 7/8

School _______________ Team Name _______________

Calculators are allowed.

1. What is the side length of a square with the same area as a triangle with base 10 and height 6? Enter your answer as a decimal rounded to three decimal places.

(20 pts) 1. _________

2. What is the area of a circle with circumference 7π? Enter your answer as a decimal rounded to three places.

(20 pts) 2. _________

3. Find positive numbers r and s with $rs = 31$ and $r + s = 12$. Enter your answers as decimals rounded to three places.

(20 pts) 3. _________

4. In a class your grade is determined by the average of five exam scores. If the average of your first three exams is 73%, what must your average on the last two exams be to get at least an 80% in the class?

(20 pts) 4. _________

5. The value of a new car decreases by 10% each year. If a new car costs \$35,000, what will its value be after 5 years?

(20 pts) 5. _________

6. What is the greatest common divisor of 175 and 280?

(20 pts) 6. _________

7. Find the positive solution to $3^{x^2+1} = 9^{2x+1}$. Enter your answer as a decimal rounded to three places.

(20 pts) 7. _________

8. Suppose the price of an item is increased by 20% and then later the new price is decreased by 30%. Compared to the original price, what percentage decrease does the final price represent?

(20 pts) 8. _________

9. Solve the system of equations

$$\begin{cases} 3x + y &= 1013 \\ 5x + 2y &= 888 \end{cases}$$

(20 pts) 9. _________

10. Suppose a and b are the roots of $x^2 + 10x - 3 = 0$. What is $a^2b + ab^2$?

(20 pts) 10. _________

TOTAL POINTS _________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

TEAM TEST 2
Grades 7/8

School _______________ Team Name _______________

Calculators are **NOT** allowed.

1. The average of five weights is 13 grams. If a 7-gram weight is added, what is the average of the six weights?

(20 pts) 1. _________

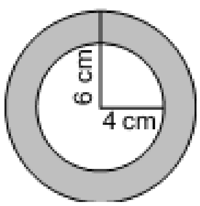
2. The 7-digit numbers $74A52B1$ and $326AB4C$ are multiples of 3. What is the largest possible value for C?

(20 pts) 2. _________

3. Every day at school, Jo climbs a flight of 6 stairs. Jo can climb using 1, 2, or 3 steps at a time or any combination of them. How many ways can Jo climb the flight of 6 stairs?

(20 pts) 3. _________

4. Two pendants are made up of the same material. They are equally thick and weigh the same. One of them has a shape of a gray "annulus" formed by two circles with radii 6 cm and 4 cm (see picture). The second has the shape of a solid circle. What is the square of the radius (i.e. radius $\times$ radius) of the second pendant?



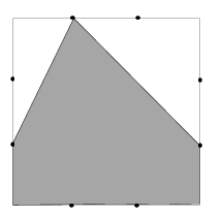
(20 pts) 4. _________

5. If $A = \frac{1}{3}, B = 5, C = 1, D = \frac{2}{3}, E = \frac{3}{2}$, $F = -1$, find the value of the expression:

$$A \cdot \frac{B + C}{D \cdot E} + F$$

(20 pts) 5. _________

6. Each side of a square is trisected at the points shown. What part of the square is shaded? Express your answer as a fraction in simplest terms.



(20 pts) 6. _________

7. There are 93 seventh graders and 108 eighth graders entering a raffle. In each grade, the number of cat owners is twice the number of students who do not own a cat. What is the probability that a seventh grader who does not own a cat wins the raffle? Express your answer as a common fraction.

(20 pts) 7. _________

8. On Friday, Pat bought some cupcakes. On Saturday, Pat gave $\frac{1}{2}$ of the cupcakes to Ryan. On Sunday, Pat gave $\frac{1}{3}$ of the remaining cupcakes to Alice. If Pat has 24 cupcakes left and no other cupcakes were given away or eaten, how many cupcakes did Pat buy on Friday?

(20 pts) 8. _________

9. What is the sum of the three smallest distinct positive integers that are both a multiple of 5 and also 1 more than a multiple of 7?

(20 pts) 9. _________

10. Maria runs twice as fast as she walks. It takes 40 minutes for her to walk from her home to school in the morning. She then runs from school to her friend's house in the afternoon. If her friend lives three times as far from the school as Maria does, how many minutes does Maria spend running in the afternoon?

(20 pts) 10. _________

TOTAL POINTS _________

Grades 7 & 8 Keys and Solutions

UND MATHEMATICS TRACK MEET — INDIVIDUAL TEST 1
University of North Dakota — Grades 7/8
January 12, 2026

School ______________ Team Name ______________

Calculators are allowed. **Key** Student Name ______________

1. Simplify $(8-5)\times[(7+4)-(6\div 3)]$. Your answer should be an integer or a fraction where the numerator and denominator have no common factors.

   (2 pts) 1. 27

2. Each student in a class has a dog, a cat, or both a dog and a cat. Sixteen students have a dog (with or without a cat), and 10 have a cat (with or without a dog). If there are 21 students in all, how many have both a dog and a cat?

   (3 pts) 2. 5

3. The 21 students in a morning math class have an average score of 86 on a test. The 18 students in the afternoon class have an average score of 73. What is the average score of all 39 students together?

   (3 pts) 3. 80

4. A rectangular prism is 8 cm long, 5 cm wide, and 4 cm high. What is the total surface area of all faces of the prism?

   (3 pts) 4. 184 cm^2

5. A line passes through three points (1, 2), (7, 4), and (16, a). What is the value of a?

   (3 pts) 5. 7

6. A box contains 3 red balls and 3 blue balls. Alice randomly draws and keeps one of the balls from the box, and then Bill randomly draws a different ball from those left in the box. What is the probability that Alice and Bill both draw red balls? Express your answer as a fraction where the numerator and denominator have no common factors.

   (3 pts) 6. $\frac{1}{5}$

7. A square has sides of length 5 inches. A circle has a diameter of 6 inches. How much longer (in inches) is the perimeter of the square than the circumference of the circle? Use 3.14 for pi, and write your answer to the hundredth place.

   (3 pts) 7. 1.16 in

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 1
Grades 7/8

School _______________ Team Name _______________

Calculators are allowed. **Solutions** Student Name _______________

1. Simplify $(8-5)\times[(7+4)-(6\div3)]$. Your answer should be an integer or a fraction where the numerator and denominator have no common factors.

(2 pts) 1. 27

Solution: $(8-5)\times[(7+4)-(6\div3)] = 3\times[11-2] = 3\times 9 = 27$

2. Each student in a class has a dog, a cat, or both a dog and a cat. Sixteen students have a dog (with or without a cat), and 10 have a cat (with or without a dog). If there are 21 students in all, how many have both a dog and a cat?

(3 pts) 2. 5

Solution: Sixteen students have a dog, so $21-16=5$ must have a cat with no dog. Since 10 have a cat, there must be $10-5=5$ with both. Or use set notation with A = the set of students with dogs and B = the set of students with cats. $|A\cup B| = |A|+|B|-|A\cap B| \Rightarrow 21 = 16+10-|A\cap B| \Rightarrow |A\cap B| = 16+10-21 = 5$

3. The 21 students in a morning math class have an average score of 86 on a test. The 18 students in the afternoon class have an average score of 73. What is the average score of all 39 students together?

(3 pts) 3. 80

Solution: The total of the 39 scores is $(21*86+18*73) = 3120$, so the average is $3120/39 = 80$.

4. A rectangular prism is 8 cm long, 5 cm wide, and 4 cm high. What is the total surface area of all faces of the prism?

(3 pts) 4. 184 cm^2

Solution: There are two faces consisting of rectangles with each possible pair of edges, so the total surface area is $2\times8\times5+2\times8\times4+2\times5\times4 = 184$ cm^2.

5. A line passes through three points (1, 2), (7, 4), and (16, a). What is the value of a?

(3 pts) 5. 7

Solution: The slope of the line is $(4-2)/(7-1) = 1/3$ and the y-intercept is $2-(1/3)*1 = 5/3$. Therefore, $a = (1/3)*16+5/3 = 21/3 = 7$.

6. A box contains 3 red balls and 3 blue balls. Alice randomly draws and keeps one of the balls from the box, and then Bill randomly draws a different ball from those left in the box. What is the probability that Alice and Bill both draw red balls? Express your answer as a fraction where the numerator and denominator have no common factors.

(3 pts) 6. $\frac{1}{5}$

Solution: The probability that Alice draws a red ball is $\frac{3}{6}$, and the probability that Bill draws a red ball after Alice has already removed a red ball is $\frac{2}{5}$. The probability that both happen is $\left(\frac{3}{6}\right)\times\left(\frac{2}{5}\right) = \frac{1}{5}$.

7. A square has sides of length 5 inches. A circle has a diameter of 6 inches. How much longer (in inches) is the perimeter of the square than the circumference of the circle? Use 3.14 for pi, and write your answer to the hundredth place.

(3 pts) 7. 1.16 in

Solution: The perimeter of the square is $4 \times 5 = 20$ in. The circumference of the circle is $6\pi = 6 \times 3.14 = 18.84$ in. The difference is $(20 - 18.84) = 1.16$ in.

UND MATHEMATICS TRACK MEET | INDIVIDUAL TEST 2
University of North Dakota | Grades 7/8
January 12, 2026

School _______________ Team Name _______________

Calculators are **NOT** allowed. **Key** Student Name _______________

1. Simplify the expression

$$\left(\frac{2}{3}\right)^2 - \left(-\frac{3}{5}\right)^2 \div \frac{27}{25}$$

(2 pts) 1. 1/9

2. Simplify the expression

$$\sqrt{24} - 7\sqrt{54} + 6\sqrt{6}$$

(3 pts) 2. $-13\sqrt{6}$

3. Factor completely over reals

$$16x^5 - 81x$$

(3 pts) 3. $x(4x^2+9)$ $(2x+3)(2x-3)$

4. Solve for x and write your answer in set notation, i.e. { }.

$$x - 4x^{1/2} = 0$$

(3 pts) 4. $\{0, 16\}$

5. Solve the inequality $3 < 2x + 2 \leq 6$.

(3 pts) 5. $\left(\frac{1}{2}, 2\right]$ or $\frac{1}{2} < x \leq 2$

6. A die is tossed. What is the probability of tossing a 2 or a 5?

(3 pts) 6. $\frac{1}{3}$

7. In 1990, Mary had a rectangular garden that measured a width of W ft and length of 20 ft. In the year 2000, she added 2 ft to the width of her flower garden. In 2010, she added another 3 ft to the width of her garden. What is the change in area from 1990 to 2010?

(3 pts) 7. 100 sq.ft

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 3
Grades 7/8

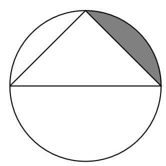
School ______________ Team Name ______________

Calculators are allowed. **Key** Student Name ______________

1. Two fair 6-sided dice are rolled and their product is recorded. Find the probability that their product is a multiple of 3. Round your answer to two decimal places.

(2 pts) 1. 55.56% or 0.56

2. An isosceles right triangle is inscribed in a circle as shown below. Given that the area of the circle is 9π, find the area of the shaded region. Round your answer to two decimal places.



(3 pts) 2. 2.57

3. Suppose that the population of a bacterial colony is currently 1248. If the population shrinks to half of its size every hour, how many hours will it be until the population reaches 156?

(3 pts) 3. 3 hours

4. Quinn invested \$500 in a fund which is growing at a rate of 3.5% per month and \$1000 in a fund which is shrinking at a rate of 2% per month. By how much did Quinn's investment change after one month?

(3 pts) 4. −\$2.50 or decrease by \$2.50

5. Find the set of all x for which $(2^x - 3)^2 + (2^x - 3) = 2$.

(3 pts) 5. $\{0, 2\}$

6. Suppose the date of the third Friday of October is two times the date of the second Tuesday of October. What day of the week is October 5th?

(3 pts) 6. Thursday

7. Starting with the number 5/7, we will begin repeating the following process. If the number is bigger than 1, subtract 1. Otherwise we multiply it by 2. Then repeat this again with the new number. How many repetitions do we make before we arrive back at 5/7?

(3 pts) 7. 5

UND MATHEMATICS TRACK MEET | INDIVIDUAL TEST 3
University of North Dakota | Grades 7/8
January 12, 2026

School _______________ Team Name _______________

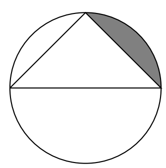
Calculators are allowed. **Solutions** Student Name _______________

1. Two fair 6-sided dice are rolled and their product is recorded. Find the probability that their product is a multiple of 3. Round your answer to two decimal places.

(2 pts) 1. 55.56% or 0.56

Solution: In order for the product to be a multiple of three, then at least one of the die must be either a 3 or a 6. Looking at the possibilities, we have: (1,1), (1,2), **(1,3)**, (1,4), (1,5), **(1,6)**, (2,1), (2,2), **(2,3)**, (2,4), (2,5), **(2,6)**,**(3,1)**, **(3,2)**, **(3,3)**, **(3,4)**, **(3,5)**, **(3,6)**, (4,1),(4,2), **(4,3)**, (4,4), (4,5), **(4,6)**, (5,1), (5,2), **(5,3)**, (5,4), (5,5), **(5,6)**, **(6,1)**, **(6,2)**,**(6,3)**, **(6,4)**,**(6,5)**, **(6,6)**. Of these, 20 of the 36 possible outcomes are multiples of 3. Therefore, the probability is $20/36 = 5/9$.

2. An isosceles right triangle is inscribed in a circle as shown below. Given that the area of the circle is 9π, find the area of the shaded region. Round your answer to two decimal places.



(3 pts) 2. 2.57

Solution: Notice that the shaded region is the same as the top right fourth of the circle minus the right half of the triangle. Furthermore, since the circle has area 9π, then its radius must be $r = 3$. Also, then the base of the triangle is $2r = 2 \cdot 3 = 6$ and the height is $r = 3$. Thus, the shaded region has area $\frac{1}{4}(9\pi) - \frac{1}{2}\left(\frac{1}{2}(6)(3)\right) = \frac{9\pi}{4} - \frac{9}{2} \approx 2.57$

3. Suppose that the population of a bacterial colony is currently 1248. If the population shrinks to half of its size every hour, how many hours will it be until the population reaches 156?

(3 pts) 3. 3 hours

Solution: After the first hour, the colony has population 624. After the second, 312. After the third, 156. Therefore, the answer is 3 hours.

4. Quinn invested \$500 in a fund which is growing at a rate of 3.5% per month and \$1000 in a fund which is shrinking at a rate of 2% per month. By how much did Quinn's investment change after one month?

(3 pts) 4. $-\$2.50$ or decrease by \$2.50

Solution: The \$500 investment increases by $(500)(0.035) = 17.5$ dollars. The \$1000 investment decreases by $(1000)(0.02) = 20$ dollars. Therefore, the total change is $17.5 - 20 = -2.5$ dollars.

5. Find the set of all x for which $(2^x - 3)^2 + (2^x - 3) = 2$.

(3 pts) 5. $\{0, 2\}$

Solution: Subtracting 2 from both sides we have $(2^x - 3)^2 + (2^x - 3) - 2 = 0$. Notice that the left-hand side factors giving us $\Big((2^x - 3) + 2\Big)\Big((2^x - 3) - 1\Big) = 0$ or more simply $(2^x - 1)(2^x - 4) = 0$. Setting each factor equal to zero, we have that either $(2^x - 1) = 0$ or $(2^x - 4) = 0$. Adding the constants to the other side, we have $2^x = 1$ or $2^x = 4$. Applying the logarithm base 2 to each side, we have $x = 0$ or $x = 2$. Therefore, the set of all x which satisfy the equation is $\{0, 2\}$.

6. Suppose the date of the third Friday of October is two times the date of the second Tuesday of October. What day of the week is October 5th?

(3 pts) 6. Thursday

Solution: Let x be the date of the first Friday of October. Then the date of the third Friday can be expressed as $x + 14$. The date of the second Tuesday would then be either $(x - 3) + 7$ or $(x + 4) + 7$. Thus we have that $x + 14 = 2\Big((x - 3) + 7\Big)$ or $x + 14 = 2\Big((x + 4) + 7\Big)$. Simplifying we have $x + 14 = 2x + 8$ or $x + 14 = 2x + 22$. Solving for x we either have $x = 6$ or $x = -4$. Since dates do not admit negative values, we have that the date of the first Friday of October is $x = 6$. Thus, October 5th would be Thursday.

7. Starting with the number 5/7, we will begin repeating the following process. If the number is bigger than 1, subtract 1. Otherwise we multiply it by 2. Then repeat this again with the new number. How many repetitions do we make before we arrive back at 5/7?

(3 pts) 7. 5

Solution: Starting with 5/7, since it is less than 1, we double it to 10/7. Then, since 10/7 is greater than 1, we subtract 1 to get 3/7. We continue this. And in order our numbers become: $\frac{5}{7} \to \frac{10}{7} \to \frac{3}{7} \to \frac{6}{7} \to \frac{12}{7} \to \frac{5}{7}$. Thus, there were 5 repetitions of this process to return to $\frac{5}{7}$.

School _______________ Team Name _______________

Calculators are **NOT** allowed. **Key** Student Name _______________

1. Evaluate $(-100)^0 + 3 \mid -4^2 - (-8) \mid +2 \mid \sqrt{16} + (-6)^2 \mid$

(2 pts) 1. 105

2. A chess club has only ninth-grade and tenth-grade students. If $\frac{3}{7}$ of the students are in ninth grade and there are 18 more tenth-grade students than ninth-grade students, how many tenth-grade students are in the club?

(3 pts) 2. 72 students

3. Two fair six-sided dice are rolled. What is the probability that the sum of the two numbers is at most 5?

(3 pts) 3. $\frac{10}{36} = \frac{5}{18}$

4. A rectangular storage container is 30 feet long, 15 feet wide, and 10 feet deep. How many gallons of paint are needed to paint the outside of the four walls and the bottom if one gallon covers 150 square feet?

(3 pts) 4. 9 gallons

5. A bakery uses 4 mixing machines. Each machine mixes 18 batches of dough all together every $\frac{3}{4}$ minute. How many batches can all 4 machines mix in 2 minutes?

(3 pts) 5. 192 batches

6. Find the x-intercept that is less than 0, if any, for

$$f(x) = (x-3)^2 - 9$$

(3 pts) 6. None

7. Simplify the expression. Write your answer as a fraction in simplified form.

$$\frac{5^3 \cdot 25^2}{125^3} \times \left(\frac{4^2 \cdot 64}{8^4}\right)^{\frac{1}{2}}$$

(3 pts) 7. $\frac{1}{50}$

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

TEAM TEST 1
Grades 7/8

School _______________

Team Name _______________

Calculators are allowed.

Key

1. What is the side length of a square with the same area as a triangle with base 10 and height 6? Enter your answer as a decimal rounded to three decimal places.

(20 pts) 1. 5.477

2. What is the area of a circle with circumference 7π? Enter your answer as a decimal rounded to three places.

(20 pts) 2. 38.485

3. Find positive numbers r and s with $rs = 31$ and $r + s = 12$. Enter your answers as decimals rounded to three places.

(20 pts) 3. $\{r, s\} \approx \{3.764, 8.236\}$

4. In a class your grade is determined by the average of five exam scores. If the average of your first three exams is 73%, what must your average on the last two exams be to get at least an 80% in the class?

(20 pts) 4. 90.5%

5. The value of a new car decreases by 10% each year. If a new car costs $35,000, what will its value be after 5 years?

(20 pts) 5. $20667.15

6. What is the greatest common divisor of 175 and 280?

(20 pts) 6. 35

7. Find the positive solution to $3^{x^2+1} = 9^{2x+1}$. Enter your answer as a decimal rounded to three places.

(20 pts) 7. 4.236

8. Suppose the price of an item is increased by 20% and then later the new price is decreased by 30%. Compared to the original price, what percentage decrease does the final price represent?

(20 pts) 8. 16%

9. Solve the system of equations

$$\begin{cases} 3x + y &= 1013 \\ 5x + 2y &= 888 \end{cases}$$

(20 pts) 9. $(x, y) = (1138, -2401)$

10. Suppose a and b are the roots of $x^2 + 10x - 3 = 0$. What is $a^2b + ab^2$?

(20 pts) 10. 30

UND MATHEMATICS TRACK MEET | TEAM TEST 1
University of North Dakota | Grades 7/8
January 12, 2026

School ______________ Team Name ______________

Calculators are allowed. **Solutions**

1. What is the side length of a square with the same area as a triangle with base 10 and height 6? Enter your answer as a decimal rounded to three decimal places.

(20 pts) 1. 5.477

Solution: We have $s^2 = \frac{1}{2} \cdot 10 \cdot 6 = 30$, and so $s = \sqrt{30} \approx 5.477$.

2. What is the area of a circle with circumference 7π? Enter your answer as a decimal rounded to three places.

(20 pts) 2. 38.485

Solution: Since $2\pi r = 7\pi$, we have $r = 7/2$, and so $\pi r^2 = \frac{49\pi}{4} \approx 38.485$

3. Find positive numbers r and s with $rs = 31$ and $r + s = 12$. Enter your answers as decimals rounded to three places.

(20 pts) 3. $\{r, s\} \approx \{3.764, 8.236\}$

Solution: Since $(x - r)(x - s) = x^2 - 12x + 31$, the quadratic formula gives $\{r, s\} \approx \{3.764, 8.236\}$.

4. In a class your grade is determined by the average of five exam scores. If the average of your first three exams is 73%, what must your average on the last two exams be to get at least an 80% in the class?

(20 pts) 4. 90.5%

Solution: If a, b, c are the first three exam scores and d, e are the last two, we have $(a + b + c)/3 = 73$ and $(a + b + c + d + e)/5 = 80$. Solve to get $d + e = 181$, and so $(c + d)/2 = 90.5$.

5. The value of a new car decreases by 10% each year. If a new car costs \$35,000, what will its value be after 5 years?

(20 pts) 5. \$20667.15

Solution: $\$35000 \cdot (.9)^5 = \$20667.15$.

6. What is the greatest common divisor of 175 and 280?

(20 pts) 6. 35

Solution: We have $175 = 7 \cdot 5^2$ and $280 = 2^3 \cdot 5 \cdot 7$ so the GCD is $5 \cdot 7 = 35$.

7. Find the positive solution to $3^{x^2+1} = 9^{2x+1}$. Enter your answer as a decimal rounded to three places.

(20 pts) 7. 4.236

Solution: We have $3^{x^2+1} = 3^{4x+2}$, and so $x^2 - 4x - 1 = 0$. The solutions are $2 \pm \sqrt{5}$. The positive solution is $2 + \sqrt{5} \approx 4.236$.

8. Suppose the price of an item is increased by 20% and then later the new price is decreased by 30%. Compared to the original price, what percentage decrease does the final price represent?

(20 pts) 8. 16%

Solution: If P is the original price, the final price is $(.7)(1.2)P = (.84)P$. So the price has decreased by 16%.

9. Solve the system of equations

$$\begin{cases} 3x + y &= 1013 \\ 5x + 2y &= 888 \end{cases}$$

(20 pts) 9. $(x, y) =$ $(1138, -2401)$

Use Gaussian elimination or substitution to find $x = 1138$, $y = -2401$.

10. Suppose a and b are the roots of $x^2 + 10x - 3 = 0$. What is $a^2b + ab^2$?

(20 pts) 10. 30

Solution: We have $a^2b + ab^2 = ab(a + b) = (-3) \cdot (-10) = 30$. Alternatively, a and b could be calculated directly, etc.

UND MATHEMATICS TRACK MEET — TEAM TEST 2

University of North Dakota — Grades 7/8

January 12, 2026

School _______________ Team Name _______________

Calculators are **NOT** allowed. **Key**

1. The average of five weights is 13 grams. If a 7-gram weight is added, what is the average of the six weights?

(20 pts) 1. 12

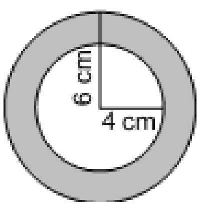
2. The 7-digit numbers $74A52B1$ and $326AB4C$ are multiples of 3. What is the largest possible value for C?

(20 pts) 2. 7

3. Every day at school, Jo climbs a flight of 6 stairs. Jo can climb using 1, 2, or 3 steps at a time or any combination of them. How many ways can Jo climb the flight of 6 stairs?

(20 pts) 3. 24

4. Two pendants are made up of the same material. They are equally thick and weigh the same. One of them has a shape of a gray "annulus" formed by two circles with radii 6 cm and 4 cm (see picture). The second has the shape of a solid circle. What is the square of the radius (i.e. radius × radius) of the second pendant?



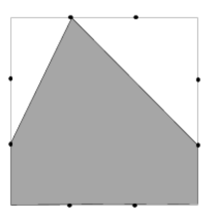
(20 pts) 4. 20

5. If $A = \frac{1}{3}, B = 5, C = 1, D = \frac{2}{3}, E = \frac{3}{2}, F = -1$, find the value of the expression:

$$A \cdot \frac{B+C}{D \cdot E} + F$$

(20 pts) 5. 1

6. Each side of a square is trisected at the points shown. What part of the square is shaded? Express your answer as a fraction in simplest terms.



(20 pts) 6. $\frac{2}{3}$

7. There are 93 seventh graders and 108 eighth graders entering a raffle. In each grade, the number of cat owners is twice the number of students who do not own a cat. What is the probability that a seventh grader who does not own a cat wins the raffle? Express your answer as a common fraction.

(20 pts) 7. $\frac{31}{201}$

8. On Friday, Pat bought some cupcakes. On Saturday, Pat gave $\frac{1}{2}$ of the cupcakes to Ryan. On Sunday, Pat gave $\frac{1}{3}$ of the remaining cupcakes to Alice. If Pat has 24 cupcakes left and no other cupcakes were given away or eaten, how many cupcakes did Pat buy on Friday?

(20 pts) 8. 72

9. What is the sum of the three smallest distinct positive integers that are both a multiple of 5 and also 1 more than a multiple of 7?

(20 pts) 9. 150

10. Maria runs twice as fast as she walks. It takes 40 minutes for her to walk from her home to school in the morning. She then runs from school to her friend's house in the afternoon. If her friend lives three times as far from the school as Maria does, how many minutes does Maria spend running in the afternoon?

(20 pts) 10. 60

Grades 9 & 10 Tests

UND MATHEMATICS TRACK MEET | INDIVIDUAL TEST 1
University of North Dakota | Grades 9/10
January 12, 2026

School ______________ Team Name ______________

Calculators are allowed. Student Name ______________

1. Roxanna has a collection of silver spoons from all over the world. She finds that she can arrange her spoons in sets of 7 with 6 left over, sets of 8 with 1 left over, or sets of 15 with 3 left over. If Roxanna has fewer than 220 spoons, how many are there?

(2 pts) 1. ________

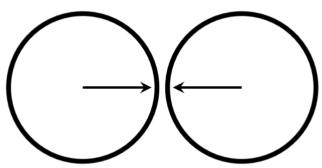
2. If any two points determine a line, how many lines are determined by seven points in a plane, no three of which are collinear?

(3 pts) 2. ________

3. A hockey team played 5 games with an average of 3 goals per game. How many goals do they need to score in their 6th game to increase their average to 4 goals per game?

(3 pts) 3. ________

4. Two circular dials are positioned next to each other, and an arrow is drawn on each dial, as shown



The left dial rotates counterclockwise at 20° per second and the right dial rotates clockwise at 8° per second. What is the minimum number of seconds that must pass before the arrows are pointing directly towards each other again?

(3 pts) 4. ________

5. Two cylinders are standing on a flat table. Cylinder A has radius 2 and height 8. Cylinder B has radius 8 and height 2. Cylinder A is $\frac{3}{4}$ full of water and Cylinder B is empty. If all of the water from Cylinder A is then poured into Cylinder B what fraction of Cylinder B is full of water? (The volume of a cylinder with radius r and height h is $V = \pi r^2 h$)

(3 pts) 5. ________

6. Given that x, y, and z are natural numbers and $xy = 8$ and $yz = 4$. What is the smallest possible value of $x + y + z$?

(3 pts) 6. ________

7. Matthew reads 3 more pages each day than the previous day. If he reads 6 pages on the first day, how many days will it take him to finish a 510-page book?

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 2
Grades 9/10

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Student Name ______________

1. What is the sum of the roots of the equation $-3x + 1 = 2x^2 + 5x + 7$?

(2 pts) 1. ________

2. If $-6 < x < 12$ and $-3 < y < -2$, then $a < x/y < b$. What is $a + b$?

(3 pts) 2. ________

3. Solve $\dfrac{2}{|x-1|} > \dfrac{1}{3}$

(3 pts) 3. ________

4. Find an equation for the set of all points (x, y) that are equidistant from $(0, 0)$ and $(3, 1)$.

(3 pts) 4. ________

5. For which value of k is $x^3 + 2x^2 + 3kx + 1$ divisible by $x - 1$?

(3 pts) 5. ________

6. If $x < -4$, then $\big|3 - |x+1|\big|$ is
(a) $x + 1$ (b) $x + 4$ (c) $-x - 1$ (d) $-x - 4$ (e) None of the above

(3 pts) 6. ________

7. A sequence is defined by $a_1 = 1, a_2 = \sqrt{3}, a_3 = \sqrt{3\sqrt{3}}, \cdots, a_n = \sqrt{3a_{n-1}}$. This sequence converges to L. What is L?

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 3
Grades 9/10

School ______________

Team Name ______________

Calculators are allowed.

Student Name ______________

1. Susan has ordered a package and lives 7 miles east and 24 miles north of the fulfillment center. The delivery drone flies in a straight line above the buildings at 50 miles per hour. How long will it take Susan to get her package in minutes?

(2 pts) 1. ________

2. A new public policy goal states that the number of high school graduates pursuing STEM careers in North Dakota must increase by 45 students per year. If the target cohort size is 2026 students in the year 2026, how many students must the policy cohort include in the baseline year 2012?

(3 pts) 2. ________

3. Let x and y be positive integers such that $x^2 + y^2 = 2026$ and $x^2y^2 = 2025$. What is $|x - y|$?

(3 pts) 3. ________

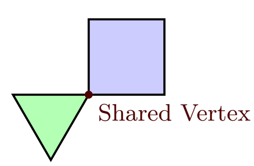
4. A standard deck of 52 cards has 13 values in each of 4 different suits. I draw two cards from the deck that are both aces and do not replace back in the deck. What is the probability that the next card I draw is an ace? Give your answers as a reduced fraction.

(3 pts) 4. ________

5. Kanesha just had a birthday. Now her daughter is half as old as her and her mother is twice as old as her. The sum of all of their ages is 161. How old did Kanesha just turn?

(3 pts) 5. ________

6. A composite animal pen is made up of a square and an equilateral triangle that meet at exactly one vertex (see picture below). Let x be the length of the side of the square, and the length of the side of the triangle is $\frac{2}{3}x$. The total perimeter of the pen is 78 units, what is the perimeter of the triangle alone?



(3 pts) 6. ________

7. A new smoothie bar is opening, and the menu offers a fixed list of four premium add-ins: Açaí, Chia Seeds, Protein Powder, and Honey. The manager wants to feature every single combination of add-ins that a customer could order: no add-ins, one add-in all the way to all four add-ins. How many total distinct menu options, representing every possible combination of add-ins, must the manager prepare prices for?

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 4
Grades 9/10

School _______________

Team Name _______________

Calculators are **NOT** allowed.

Student Name _______________

1. In a survey, 130 students were asked if they liked dogs and if they liked cats. A total of 90 students like dogs. A total of 67 like cats. A total of 13 like neither cats nor dogs. How many of the 130 students like both cats and dogs?

(2 pts) 1. _________

2. A 24-hr digital clock shows a time of 12:34. How many minutes will pass until the clock next shows a time in which all of the digits are consecutive and are in increasing order?

(3 pts) 2. _________

3. Your car is currently 3 years old and is worth \$10,000. Three years ago, it was worth \$13,000. Assume the car's value depreciates linearly with time. What will be the value of the car in 5 years?

(3 pts) 3. _________

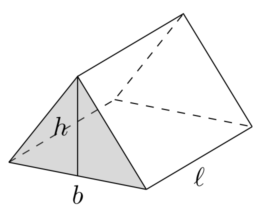
4. An urn is filled with coins and beads, all of which are either silver or gold. Ninety percent of the objects in the urn are beads. Sixty percent of the coins in the urn are silver. What percent of the objects in the urn are gold coins?

(3 pts) 4. _________

5. A number is multiplied by seven and then sixty-two is subtracted from the result to give ninety-nine. What is the original number?

(3 pts) 5. _________

6. The triangular prism shown below has a volume of 10. A new prism is formed by doubling the base and height of the original triangle, and tripling the length of the original prism. What is the volume of the new prism?



(3 pts) 6. _________

7. Real numbers a and b satisfy the equations $3^a = 27^{b+1}$ and $32^b = 2^{2(a-4)}$. What is ab?

(3 pts) 7. _________

TOTAL POINTS _________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

TEAM TEST 1
Grades 9/10

School ______________ Team Name ______________

Calculators are allowed.

1. Determine exactly the value of x for which $\dfrac{3x-5}{2} = \dfrac{5}{4}$.

(20 pts) 1. ________

2. If a and b are positive real numbers and $\frac{a^2+b^2}{\frac{1}{a^2}+\frac{1}{b^2}} = 10$, determine exactly the value of $\frac{a^3+b^3}{\frac{1}{a^3}+\frac{1}{b^3}}$.

(20 pts) 2. ________

3. a, b, and c are positive integers such that the sum of a and b is 3 more than the value of c, the value of a is double that of b, and the value of b is five less than c. Determine exactly the value of b.

(20 pts) 3. ________

4. The base of a triangular flower bed is 4 meters longer than its height. If the area of the flower bed is 30 m^2, find the height of the triangle.

(20 pts) 4. ________

5. Determine exactly the coordinates of the intersection of the lines $x + y = 15$ and $5x + 8y = 87$.

(20 pts) 5. ________

6. Buddy delivers the same number of local newspapers every day during the summer. He gets paid \$0.25 for each paper delivered, except on Sundays, when he gets paid \$1.00 per paper. After three full weeks, Buddy has earned \$900. how many papers does he deliver daily?

(20 pts) 6. ________

7. Five years ago, Maria was three times as old as Alex. Five years from now, Maria will be twice as old as Alex. What is Alex's current age?

(20 pts) 7. ________

8. Find the constants A, B, and C such that $\dfrac{A}{(x-2)(x-4)} + \dfrac{B}{x-2} + \dfrac{C}{x(x-4)} = \dfrac{1}{x}$ for all permissible x.

(20 pts) 8. ________

9. (x, y) is the intersection of $\dfrac{3x}{y} - 4 = 11$ and $x - y = 44$. Determine the value of $x + y$.

(20 pts) 9. ________

10. How many pairs of positive integers (x, y), where $x + y \le 2026$, satisfy the equation $\dfrac{x + y^{-1}}{x^{-1} + y} = 11$?

(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

TEAM TEST 2
Grades 9/10

School _______________ Team Name _______________

Calculators are **NOT** allowed.

1. A sequence is defined by $a_1 = 4$ and $a_{n+1} = a_n + 3$. What is a_{20}?

(20 pts) 1. _________

2. What is the remainder when 7^{2025} is divided by 6?

(20 pts) 2. _________

3. The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5050 is collected. How many more children than adults attended?

(20 pts) 3. _________

4. A right triangle has legs 6 and 8. A square is drawn with the hypotenuse of the right triangle as a side. What is the area of the square?

(20 pts) 4. _________

5. A rectangle's diagonal is 25 and one side is 7. What is the perimeter of the rectangle?

(20 pts) 5. _________

6. How many distinct ways can the letters in the word "LEVEL" be arranged?

(20 pts) 6. _________

7. A school has 5 math teams and each team has 4 students. How many students must you choose to guarantee that at least two are from the same team?

(20 pts) 7. _________

8. If the cost of a bat and a baseball combined is \$1.10 and the ball costs \$1.00 less than the bat, how much is the ball?

(20 pts) 8. _________

9. Solve for x: $\sqrt{x+9} - \sqrt{x-7} = 2$.

(20 pts) 9. _________

10. In how many ways can 6 students sit in a row if two particular students insist on sitting next to each other?

(20 pts) 10. _________

TOTAL POINTS _________

Grades 9 & 10 Keys and Solutions

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 1
Grades 9/10

School ______________ Team Name ______________

Calculators are allowed. **Key** Student Name ______________

1. Roxanna has a collection of silver spoons from all over the world. She finds that she can arrange her spoons in sets of 7 with 6 left over, sets of 8 with 1 left over, or sets of 15 with 3 left over. If Roxanna has fewer than 220 spoons, how many are there?

(2 pts) 1. 153

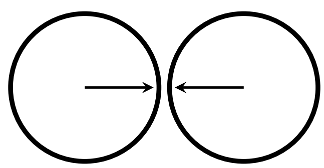
2. If any two points determine a line, how many lines are determined by seven points in a plane, no three of which are collinear?

(3 pts) 2. 21

3. A hockey team played 5 games with an average of 3 goals per game. How many goals do they need to score in their 6th game to increase their average to 4 goals per game?

(3 pts) 3. 9

4. Two circular dials are positioned next to each other, and an arrow is drawn on each dial, as shown



The left dial rotates counterclockwise at 20° per second and the right dial rotates clockwise at 8° per second. What is the minimum number of seconds that must pass before the arrows are pointing directly towards each other again?

(3 pts) 4. 90

5. Two cylinders are standing on a flat table. Cylinder A has radius 2 and height 8. Cylinder B has radius 8 and height 2. Cylinder A is $\frac{3}{4}$ full of water and Cylinder B is empty. If all of the water from Cylinder A is then poured into Cylinder B what fraction of Cylinder B is full of water? (The volume of a cylinder with radius r and height h is $V = \pi r^2 h$)

(3 pts) 5. $\frac{3}{16}$

6. Given that x, y, and z are natural numbers and $xy = 8$ and $yz = 4$. What is the smallest possible value of $x + y + z$?

(3 pts) 6. 7

7. Matthew reads 3 more pages each day than the previous day. If he reads 6 pages on the first day, how many days will it take him to finish a 510-page book?

(3 pts) 7. 17

UND MATHEMATICS TRACK MEET — INDIVIDUAL TEST 1

University of North Dakota — Grades 9/10

January 12, 2026

School ______________ Team Name ______________

Calculators are allowed. **Solutions** Student Name ______________

1. Roxanna has a collection of silver spoons from all over the world. She finds that she can arrange her spoons in sets of 7 with 6 left over, sets of 8 with 1 left over, or sets of 15 with 3 left over. If Roxanna has fewer than 220 spoons, how many are there?

   (2 pts) 1. 153

   The numbers 3 greater than a multiple of 15 are: 18, 33, 48, 63, 78, 93, 108, 123, 138, 153, 168, 183, 198, and 213. Of these, the ones that are 1 greater than a multiple of 8 are 33 and 153. Only 153 is also 6 greater than a multiple of 7.

2. If any two points determine a line, how many lines are determined by seven points in a plane, no three of which are collinear?

   (3 pts) 2. 21

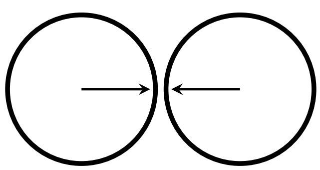
   $6 \cdot 7 = 42$, but order doesn't matter, so half of that is 21.

3. A hockey team played 5 games with an average of 3 goals per game. How many goals do they need to score in their 6th game to increase their average to 4 goals per game?

   (3 pts) 3. 9

   They've scored 15 goals so far. If they score x goals in their 6th game, their average will be $\frac{15+x}{6}$. This is 4 when $x = 9$.

4. Two circular dials are positioned next to each other, and an arrow is drawn on each dial, as shown



   The left dial rotates counterclockwise at 20° per second and the right dial rotates clockwise at 8° per second. What is the minimum number of seconds that must pass before the arrows are pointing directly towards each other again?

   (3 pts) 4. 90

   The left dial completes a rotation every 18 seconds, while the right dial completes a rotation every 45 seconds. The least common multiple of these is 90 seconds.

5. Two cylinders are standing on a flat table. Cylinder A has radius 2 and height 8. Cylinder B has radius 8 and height 2. Cylinder A is $\frac{3}{4}$ full of water and Cylinder B is empty. If all of the water from Cylinder A is then poured into Cylinder B what fraction of Cylinder B is full of water? (The volume of a cylinder with radius r and height h is $V = \pi r^2 h$)

   (3 pts) 5. $\frac{3}{16}$

   The volume of water in A is $\pi 2^2 \cdot 8 \cdot \frac{3}{4} = 24\pi$. This is $\pi 8^2 h_B$ when $h_B = \frac{24}{64} = \frac{3}{8}$. Since the height of B is 2, B will be $\frac{3}{16}$ full.

6. Given that x, y, and z are natural numbers and $xy = 8$ and $yz = 4$. What is the smallest possible value of $x + y + z$?

(3 pts) 6. ___7___

Since y divides 4, it must be 1, 2, or 4. If $y = 1$, then $x = 8$, $z = 4$, and $x+y+z = 13$. If $y = 2$, then $x = 4$, $z = 2$, and $x + y + z = 8$. If $y = 4$, then $x = 2$, $z = 1$, and $x + y + z = 7$. The smallest of these is 7.

7. Matthew reads 3 more pages each day than the previous day. If he reads 6 pages on the first day, how many days will it take him to finish a 510-page book?

(3 pts) 7. ___17___

On day n he reads $3 + 3n$ pages, so after N days he has read $\sum_{n=1}^{N} 3 + 3n = 3N + 3N(N + 1)/2$ pages. This equals 510 when $6N + 3N(N + 1) = 1020$, or $0 = N^2 + 3N - 340 = (N - 17)(N + 20)$, so that $N = 17$.

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 2
Grades 9/10

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Key

Student Name ______________

1. What is the sum of the roots of the equation $-3x+1=2x^2+5x+7$?

(2 pts) 1. -4

2. If $-6<x<12$ and $-3<y<-2$, then $a<x/y<b$. What is $a+b$?

(3 pts) 2. -3

3. Solve $\dfrac{2}{|x-1|}>\dfrac{1}{3}$

(3 pts) 3. $-5<x<7$, $x \neq 1$

4. Find an equation for the set of all points (x,y) that are equidistant from $(0,0)$ and $(3,1)$.

(3 pts) 4. $6x+2y=10$ or $3x+y=5$

5. For which value of k is $x^3+2x^2+3kx+1$ divisible by $x-1$?

(3 pts) 5. $-4/3$

6. If $x<-4$, then $\big|3-|x+1|\big|$ is
(a) $x+1$ (b) $x+4$ (c) $-x-1$ (d) $-x-4$ (e) None of the above

(3 pts) 6. (d)

7. A sequence is defined by $a_1=1, a_2=\sqrt{3}, a_3=\sqrt{3\sqrt{3}}, \cdots, a_n=\sqrt{3a_{n-1}}$. This sequence converges to L. What is L?

(3 pts) 7. 3

UND MATHEMATICS TRACK MEET — INDIVIDUAL TEST 2
University of North Dakota — Grades 9/10
January 12, 2026

School ______________ Team Name ______________

Calculators are **NOT** allowed. **Solutions** Student Name ______________

1. What is the sum of the roots of the equation $-3x+1=2x^2+5x+7$?

(2 pts) 1. -4

Solution: $2x^2+8x+6=0 \implies x^2+4x+3=0$. The sum is -4.

2. If $-6<x<12$ and $-3<y<-2$, then $a<x/y<b$. What is $a+b$?

(3 pts) 2. -3

Solution: $-6<x<12, -1/2<1/y<-1/3 \implies -6<x/y<3$. So, $a+b=-3$.

3. Solve $\dfrac{2}{|x-1|}>\dfrac{1}{3}$

(3 pts) 3. $-5<x<7$, $x\neq 1$

Solution: $\dfrac{|x-1|}{2}<3 \implies |x-1|<6 \implies -5<x<7, x\neq 1$.

4. Find an equation for the set of all points (x,y) that are equidistant from $(0,0)$ and $(3,1)$.

(3 pts) 4. $6x+2y=10$ or $3x+y=5$

Solution: $x^2+y^2=(x-3)^2+(y-1)^2 \implies x^2+y^2=x^2-6x+9+y^2-2y+1 \implies 6x+2y=10$.

5. For which value of k is $x^3+2x^2+3kx+1$ divisible by $x-1$?

(3 pts) 5. $-4/3$

Solution: $x^3+2x^2+3kx+1=(x-1)(x^2+3x+3k+3)+3k+4$.

6. If $x<-4$, then $\big|3-|x+1|\big|$ is
(a) $x+1$ (b) $x+4$ (c) $-x-1$ (d) $-x-4$ (e) None of the above

(3 pts) 6. (d)

Solution: $\big|3-|x+1|\big|=|3+(x+1)|=|x+4|=-(x+4)$.

7. A sequence is defined by $a_1=1, a_2=\sqrt{3}, a_3=\sqrt{3\sqrt{3}}, \cdots, a_n=\sqrt{3a_{n-1}}$. This sequence converges to L. What is L?

(3 pts) 7. 3

Solution: $L=\sqrt{3L} \implies L^2=3L \implies L(L-3)=0$. Since $L\neq 0$, $L=3$.

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 3
Grades 9/10

School ______________

Team Name ______________

Calculators are allowed.

Key

Student Name ______________

1. Susan has ordered a package and lives 7 miles east and 24 miles north of the fulfillment center. The delivery drone flies in a straight line above the buildings at 50 miles per hour. How long will it take Susan to get her package in minutes?

(2 pts) 1. 30

2. A new public policy goal states that the number of high school graduates pursuing STEM careers in North Dakota must increase by 45 students per year. If the target cohort size is 2026 students in the year 2026, how many students must the policy cohort include in the baseline year 2012?

(3 pts) 2. 1396

3. Let x and y be positive integers such that $x^2 + y^2 = 2026$ and $x^2y^2 = 2025$. What is $|x - y|$?

(3 pts) 3. 44

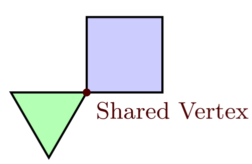
4. A standard deck of 52 cards has 13 values in each of 4 different suits. I draw two cards from the deck that are both aces and do not replace back in the deck. What is the probability that the next card I draw is an ace? Give your answers as a reduced fraction.

(3 pts) 4. $\frac{1}{25}$

5. Kanesha just had a birthday. Now her daughter is half as old as her and her mother is twice as old as her. The sum of all of their ages is 161. How old did Kanesha just turn?

(3 pts) 5. 46

6. A composite animal pen is made up of a square and an equilateral triangle that meet at exactly one vertex (see picture below). Let x be the length of the side of the square, and the length of the side of the triangle is $\frac{2}{3}x$. The total perimeter of the pen is 78 units, what is the perimeter of the triangle alone?



(3 pts) 6. 26

7. A new smoothie bar is opening, and the menu offers a fixed list of four premium add-ins: Açaí, Chia Seeds, Protein Powder, and Honey. The manager wants to feature every single combination of add-ins that a customer could order: no add-ins, one add-in all the way to all four add-ins. How many total distinct menu options, representing every possible combination of add-ins, must the manager prepare prices for?

(3 pts) 7. 16

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 3
Grades 9/10

School _______________

Team Name _______________

Calculators are allowed.

Solutions

Student Name _______________

1. Susan has ordered a package and lives 7 miles east and 24 miles north of the fulfillment center. The delivery drone flies in a straight line above the buildings at 50 miles per hour. How long will it take Susan to get her package in minutes?

(2 pts) 1. 30

Solution: The distance the drone travels can be found using the Pythagorean Theorem: $7^2 + 24^2 = 25^2$. If the drone travels 50 miles in one hour, then it will take 30 minutes to travel 25 miles.

2. A new public policy goal states that the number of high school graduates pursuing STEM careers in North Dakota must increase by 45 students per year. If the target cohort size is 2026 students in the year 2026, how many students must the policy cohort include in the baseline year 2012?

(3 pts) 2. 1396

Solution: $2026 - 2012 = 14$, so there are 14 years with an increase of 45 students each year, which is $14 * 45 = 630$, so $2026 - 630 = 1396$

3. Let x and y be positive integers such that $x^2 + y^2 = 2026$ and $x^2y^2 = 2025$. What is $|x - y|$?

(3 pts) 3. 44

Solution: $x^2y^2 = 2025 \to (xy)^2 = 2025$, so $xy = 45$. Checking all of the positive integer factors of 45, you will find that they must be 1 and 45, so the absolute value of the difference is 44

4. A standard deck of 52 cards has 13 values in each of 4 different suits. I draw two cards from the deck that are both aces and do not replace back in the deck. What is the probability that the next card I draw is an ace? Give your answers as a reduced fraction.

(3 pts) 4. $\frac{1}{25}$

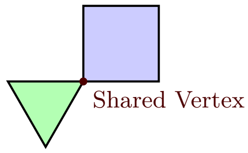
Solution: There are two aces left in the deck and a total of 50 cards, giving $\frac{2}{50}$, which reduces to $\frac{1}{25}$

5. Kanesha just had a birthday. Now her daughter is half as old as her and her mother is twice as old as her. The sum of all of their ages is 161. How old did Kanesha just turn?

(3 pts) 5. 46

Solution: Let x be Kanesha's age. Then her daughter is $.5x$ and her mother is $2x$. $2x + x + .5x = 161$, so $x = 46$.

6. A composite animal pen is made up of a square and an equilateral triangle that meet at exactly one vertex (see picture below). Let x be the length of the side of the square, and the length of the side of the triangle is $\frac{2}{3}x$. The total perimeter of the pen is 78 units, what is the perimeter of the triangle alone?



(3 pts) 6. 26

Solution: $3 * \frac{2}{3}x + 4x = 78$, so $x = 13$. The perimeter of the triangle would be $3 * \frac{2}{3} * 13 = 26$

7. A new smoothie bar is opening, and the menu offers a fixed list of four premium add-ins: Açaí, Chia Seeds, Protein Powder, and Honey. The manager wants to feature every single combination of add-ins that a customer could order: no add-ins, one add-in all the way to all four add-ins. How many total distinct menu options, representing every possible combination of add-ins, must the manager prepare prices for?

(3 pts) 7. 16

Solution: Each add-in is either in the order or not, so there are two options for each of the four add-ins. Thus $2^4 = 16$. Alternatively, you could just list all 16 options.

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 4
Grades 9/10

School _______________ Team Name _______________

Calculators are **NOT** allowed. **Key** Student Name _______________

1. In a survey, 130 students were asked if they liked dogs and if they liked cats. A total of 90 students like dogs. A total of 67 like cats. A total of 13 like neither cats nor dogs. How many of the 130 students like both cats and dogs?

(2 pts) 1. 40

2. A 24-hr digital clock shows a time of 12:34. How many minutes will pass until the clock next shows a time in which all of the digits are consecutive and are in increasing order?

(3 pts) 2. 671 min

3. Your car is currently 3 years old and is worth \$10,000. Three years ago, it was worth \$13,000. Assume the car's value depreciates linearly with time. What will be the value of the car in 5 years?

(3 pts) 3. \$5000

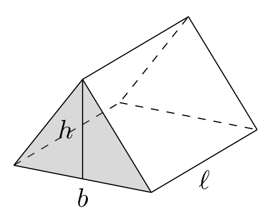
4. An urn is filled with coins and beads, all of which are either silver or gold. Ninety percent of the objects in the urn are beads. Sixty percent of the coins in the urn are silver. What percent of the objects in the urn are gold coins?

(3 pts) 4. 4%

5. A number is multiplied by seven and then sixty-two is subtracted from the result to give ninety-nine. What is the original number?

(3 pts) 5. 23

6. The triangular prism shown below has a volume of 10. A new prism is formed by doubling the base and height of the original triangle, and tripling the length of the original prism. What is the volume of the new prism?



(3 pts) 6. 120

7. Real numbers a and b satisfy the equations $3^a = 27^{b+1}$ and $32^b = 2^{2(a-4)}$. What is ab?

(3 pts) 7. 18

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 4
Grades 9/10

School _______________

Team Name _______________

Calculators are **NOT** allowed.

Solutions

Student Name _______________

1. In a survey, 130 students were asked if they liked dogs and if they liked cats. A total of 90 students like dogs. A total of 67 like cats. A total of 13 like neither cats nor dogs. How many of the 130 students like both cats and dogs?

(2 pts) 1. 40

Solution: $90 + 67 - x + 13 = 130 \implies 170 - x = 130 \implies x = 40$

2. A 24-hr digital clock shows a time of 12:34. How many minutes will pass until the clock next shows a time in which all of the digits are consecutive and are in increasing order?

(3 pts) 2. 671 min

Solution: The next time is 23:45, the amount of time gone by is 11 hours and 11 minutes. $11(60) = 660$ and $660 + 11 = 671$ minutes

3. Your car is currently 3 years old and is worth \$10,000. Three years ago, it was worth \$13,000. Assume the car's value depreciates linearly with time. What will be the value of the car in 5 years?

(3 pts) 3. \$5000

Solution: Two points (0,13000) and (3, 10000). We know the model fits this form $y = mx + b$. We need to find m and b, the initial point is given so $b = 13000$ and $m = \frac{13000-10000}{0-3} = -1000$. Then to find the value after 8 years, we have $y = -1000(8) + 13000 = 5000$

4. An urn is filled with coins and beads, all of which are either silver or gold. Ninety percent of the objects in the urn are beads. Sixty percent of the coins in the urn are silver. What percent of the objects in the urn are gold coins?

(3 pts) 4. 4%

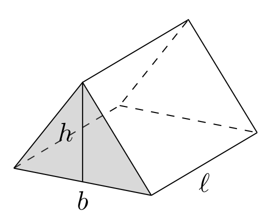
Solution: $0.9 + g + s = 1$ were g percentage of gold coins and s is silver coins.
$s = 0.1(0.6) = 0.06$, then $0.9 + g + 0.06 = 1 \implies g = 0.04$

5. A number is multiplied by seven and then sixty-two is subtracted from the result to give ninety-nine. What is the original number?

(3 pts) 5. 23

Solution: $7x - 62 = 99 \implies 7x = 161 \implies x = 23$

6. The triangular prism shown below has a volume of 10. A new prism is formed by doubling the base and height of the original triangle, and tripling the length of the original prism. What is the volume of the new prism?



(3 pts) 6. 120

Solution: $\frac{1}{2}bhl = 10$ the new prism is

$$\frac{1}{2}bhl = 10 \implies \frac{1}{2}(2b)(2h)(3l) = 12(\frac{1}{2}bhl) = 12(10) = 120$$

7. Real numbers a and b satisfy the equations $3^a = 27^{b+1}$ and $32^b = 2^{2(a-4)}$. What is ab?

(3 pts) 7. 18

Solution: Rewrite to have the same base $3^a = 3^{3(b+1)}$ and $2^{5b} = 2^{2a-8}$. Now, $a = 3(b+1)$ and $5b = 2a-8$ such that $5b = 2(3(b+1))-8 \implies 5b = 6b+6-8 \implies b = 2$. Next, $a = 3(2)+3 = 9$. Finally, $ab = 2(9) = 18$

UND MATHEMATICS TRACK MEET | TEAM TEST 1
University of North Dakota | Grades 9/10
January 12, 2026

School ______________ Team Name ______________

Calculators are allowed. **Key**

1. Determine exactly the value of x for which $\frac{3x-5}{2} = \frac{5}{4}$.

(20 pts) 1. $\frac{5}{2}$

2. If a and b are positive real numbers and $\frac{a^2+b^2}{\frac{1}{a^2}+\frac{1}{b^2}} = 10$, determine exactly the value of $\frac{a^3+b^3}{\frac{1}{a^3}+\frac{1}{b^3}}$.

(20 pts) 2. $10\sqrt{10}$

3. a, b, and c are positive integers such that the sum of a and b is 3 more than the value of c, the value of a is double that of b, and the value of b is five less than c. Determine exactly the value of b.

(20 pts) 3. 4

4. The base of a triangular flower bed is 4 meters longer than its height. If the area of the flower bed is 30 m^2, find the height of the triangle.

(20 pts) 4. 6 m

5. Determine exactly the coordinates of the intersection of the lines $x + y = 15$ and $5x + 8y = 87$.

(20 pts) 5. $(11, 4)$

6. Buddy delivers the same number of local newspapers every day during the summer. He gets paid \$0.25 for each paper delivered, except on Sundays, when he gets paid \$1.00 per paper. After three full weeks, Buddy has earned \$900. how many papers does he deliver daily?

(20 pts) 6. 120

7. Five years ago, Maria was three times as old as Alex. Five years from now, Maria will be twice as old as Alex. What is Alex's current age?

(20 pts) 7. 15

8. Find the constants A, B, and C such that $\frac{A}{(x-2)(x-4)} + \frac{B}{x-2} + \frac{C}{x(x-4)} = \frac{1}{x}$ for all permissible x.

(20 pts) 8. $A = 2$, $B = 1$, $C = -4$

9. (x, y) is the intersection of $\frac{3x}{y} - 4 = 11$ and $x - y = 44$. Determine the value of $x + y$.

(20 pts) 9. 66

10. How many pairs of positive integers (x, y), where $x + y \leq 2026$, satisfy the equation $\frac{x + y^{-1}}{x^{-1} + y} = 11$?

(20 pts) 10. 168

UND MATHEMATICS TRACK MEET — TEAM TEST 1
University of North Dakota — Grades 9/10
January 12, 2026

School _______________ Team Name _______________

Calculators are allowed.

Solutions

1. Determine exactly the value of x for which $\dfrac{3x-5}{2} = \dfrac{5}{4}$.

(20 pts) 1. $\frac{5}{2}$

Solution: Multiply both sides by 8 to obtain $4(3x-5) = 10 \implies 12x - 20 = 10 \implies 12x = 30 \implies x = \frac{30}{12} = \frac{5}{2}$.

2. If a and b are positive real numbers and $\frac{a^2+b^2}{\frac{1}{a^2}+\frac{1}{b^2}} = 10$, determine exactly the value of $\frac{a^3+b^3}{\frac{1}{a^3}+\frac{1}{b^3}}$.

(20 pts) 2. $10\sqrt{10}$

Solution: The first expression can be rewritten as $\frac{a^2+b^2}{\frac{a^2+b^2}{a^2b^2}} = 10$. Therefore, $a^2b^2 = 10 \implies ab = \sqrt{10}$. The second expressison can likewise be written as $\frac{a^3+b^3}{\frac{a^3+b^3}{a^3b^3}} = a^3b^3 = (ab)^3 = (\sqrt{10})^3 = 10\sqrt{10}$.

3. a, b, and c are positive integers such that the sum of a and b is 3 more than the value of c, the value of a is double that of b, and the value of b is five less than c. Determine exactly the value of b.

(20 pts) 3. 4

Solution: If the sum of a and b is 3 more than c, then $a + b = c + 3$. Since a is double b and b is five less than c, then $a = 2b$ and $b = c-5$ respectively. Substituting the second and third equations into the first gives us:

$$2(c-5) + (c-5) = c + 3 \Rightarrow 3(c-5) = c + 3 \Rightarrow 2c = 18 \Rightarrow c = 9$$

Therefore, $b = 9 - 5 = 4$

4. The base of a triangular flower bed is 4 meters longer than its height. If the area of the flower bed is 30 m^2, find the height of the triangle.

(20 pts) 4. 6 m

Solution: Let the height of the triangle be h. Then, the base is $h + 4$. Substitute into the formula for the area of a triangle:

$$\begin{aligned} 30 &= \frac{1}{2} \cdot (h+4) \cdot h \\ 30 &= \frac{1}{2}(h^2 + 4h) \\ 60 &= h^2 + 4h \\ 0 &= h^2 + 4h - 60 \\ 0 &= (h+10)(h-6) \end{aligned}$$

Solving for h, we get $h = -10$ and $h = 6$, but height here cannot be negative, so our height must be 6 meters.

5. Determine exactly the coordinates of the intersection of the lines $x + y = 15$ and $5x + 8y = 87$.

(20 pts) 5. $(11, 4)$

Solution: Subtracting $5x + 5y = 75$ from $5x + 8y = 87$ yields $3y = 12$ or $y = 4$. Then, $5x = 55$ and $x = 11$, so the coordinates are $(11, 4)$.

6. Buddy delivers the same number of local newspapers every day during the summer. He gets paid \$0.25 for each paper delivered, except on Sundays, when he gets paid \$1.00 per paper. After three full weeks, Buddy has earned \$900. how many papers does he deliver daily?

(20 pts) 6. 120

Solution: Let $p =$ the number of papers delivered daily. Then, Buddy earns $0.25p$ on each of six days a week, and $1.00p$ on Sundays. His weekly pay is $0.25p(6) + 1.00p = 2.50p$. Write an equation representing 3 weeks' pay: $2.50p(3) = 900 \implies 2.50p = 300 \implies p = 120$.

7. Five years ago, Maria was three times as old as Alex. Five years from now, Maria will be twice as old as Alex. What is Alex's current age?

(20 pts) 7. 15

Solution:

1. Five years ago:
   Maria's age was $y - 5$, and Alex's age was $x - 5$. From the problem:

$$y - 5 = 3(x - 5).$$

2. Five years from now:
Maria's age will be $y + 5$, and Alex's age will be $x + 5$. From the problem:

$$y + 5 = 2(x + 5).$$

3. Solve the system of equations:
From the first equation:

$$y - 5 = 3x - 15 \quad \Rightarrow \quad y = 3x - 10.$$

Substitute $y = 3x - 10$ into the second equation:

$$(3x - 10) + 5 = 2(x + 5).$$

Simplify:

$$3x - 5 = 2x + 10.$$

Solve for x:

$$x = 15.$$

Hence, Alex is currently 15 years old.

8. Find the constants A, B, and C such that $\dfrac{A}{(x-2)(x-4)} + \dfrac{B}{x-2} + \dfrac{C}{x(x-4)} = \dfrac{1}{x}$ for all permissible x.

(20 pts) 8. $A = 2$, $B = 1$, $C = -4$

Solution: Multiplying the equation by $x(x-2)(x-4)$ produces: $Ax + Bx(x-4) + C(x-2) = (x-2)(x-4)$. Expanding and simplifying yields: $Bx^2 + (A - 4B + C)x - 2C = x^2 - 6x + 8$. Therefore, $B = 1$, $A - 4B + C = -6$, and $-2C = 8$, so $C = -4$ and $A = -6 + 4 + 4 = 2$.

9. (x, y) is the intersection of $\dfrac{3x}{y} - 4 = 11$ and $x - y = 44$. Determine the value of $x + y$.

(20 pts) 9. 66

Solution: $3x = 15y \Rightarrow x = 5y$. Therefore, $5y - y = 44 \Rightarrow y = 11$. So, $x = 55$ and $x + y = 66$

10. How many pairs of positive integers (x, y), where $x + y \leq 2026$, satisfy the equation $\dfrac{x + y^{-1}}{x^{-1} + y} = 11$?

(20 pts) 10. 168

Solution: Multiply numerator and denominator by xy to obtain $\frac{x^2y+x}{y+xy^2} = \frac{x(xy+1)}{y(1+xy)} = \frac{x}{y}$. Therefore, $\frac{x}{y} = 11$, or $x = 11y$. Since $x + y \leq 2026$, $12y \leq 2026$. Therefore, $0 < y \leq 168$, and each one of these 168 values of y produces an ordered pair $(11y, y)$ that is a solution.

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

TEAM TEST 2
Grades 9/10

School _______________ Team Name _______________

Calculators are **NOT** allowed. **Key**

1. A sequence is defined by $a_1 = 4$ and $a_{n+1} = a_n + 3$. What is a_{20}?

   (20 pts) 1. 61

2. What is the remainder when 7^{2025} is divided by 6?

   (20 pts) 2. 1

3. The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5050 is collected. How many more children than adults attended?

   (20 pts) 3. 800

4. A right triangle has legs 6 and 8. A square is drawn with the hypotenuse of the right triangle as a side. What is the area of the square?

   (20 pts) 4. 100

5. A rectangle's diagonal is 25 and one side is 7. What is the perimeter of the rectangle?

   (20 pts) 5. 62

6. How many distinct ways can the letters in the word "LEVEL" be arranged?

   (20 pts) 6. 30

7. A school has 5 math teams and each team has 4 students. How many students must you choose to guarantee that at least two are from the same team?

   (20 pts) 7. 6

8. If the cost of a bat and a baseball combined is \$1.10 and the ball costs \$1.00 less than the bat, how much is the ball?

   (20 pts) 8. \$0.05

9. Solve for x: $\sqrt{x+9} - \sqrt{x-7} = 2$.

   (20 pts) 9. $x = 16$

10. In how many ways can 6 students sit in a row if two particular students insist on sitting next to each other?

    (20 pts) 10. 240

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

TEAM TEST 2
Grades 9/10

School ______________ Team Name ______________

Calculators are **NOT** allowed.

Solutions

1. A sequence is defined by $a_1 = 4$ and $a_{n+1} = a_n + 3$. What is a_{20}?

(20 pts) 1. 61

Solution: We are given $a_1 = 4$ and $a_{n+1} = a_n + 3$. This is an arithmetic sequence with first term 4 and common difference 3.

The n-th term of an arithmetic sequence is

$$a_n = a_1 + (n-1)d.$$

Here $a_1 = 4$, $d = 3$, and $n = 20$:

$$a_{20} = 4 + (20-1)\cdot 3 = 4 + 19 \cdot 3 = 4 + 57 = 61.$$

2. What is the remainder when 7^{2025} is divided by 6?

(20 pts) 2. 1

Solution: We want the remainder when 7^{2025} is divided by 6.

Notice that

$$7 \equiv 1 \pmod 6,$$

so

$$7^{2025} \equiv 1^{2025} \equiv 1 \pmod 6.$$

Therefore, the remainder is 1.

3. The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5050 is collected. How many more children than adults attended?

(20 pts) 3. 800

Solution: Let x denote the number of adults and let y denote the number of children. Then, we have the system

$$\begin{cases} x + y &= 2200 \\ 4x + 1.5y &= 5050 \end{cases}$$

So, we solve the system for x and y using whatever method seems easiest. Here, we opt for substitution and solve the first equation for x:

$$\begin{aligned} x + y &= 2200 \\ x &= 2200 - y \end{aligned}$$

Now, we substitute our formula for x in to the second equation

$$\begin{aligned} 4x + 1.5y &= 5050 \\ 4(2200 - y) + 1.5y &= 5050 \\ 8800 - 2.5y &= 5050 \\ y &= \frac{5050 - 8800}{-2.5} = 1500. \end{aligned}$$

Using our equation from the first step, this gives $x = 2200 - 1500 = 700$. So, we have our answer: 700 adults and 1500 children attended the fair, and $1500 - 700 = 800$ more children attended.

4. A right triangle has legs 6 and 8. A square is drawn with the hypotenuse of the right triangle as a side. What is the area of the square?

(20 pts) 4. 100

Solution: theorem:

$$c^2 = 6^2 + 8^2 = 36 + 64 = 100,$$

so

$$c = \sqrt{100} = 10.$$

A square on the hypotenuse has side length 10, so its area is

$$10^2 = 100.$$

5. A rectangle's diagonal is 25 and one side is 7. What is the perimeter of the rectangle?

(20 pts) 5. 62

Solution: Let the sides of the rectangle be 7 and b. The diagonal is 25. By the Pythagorean theorem,

$$7^2 + b^2 = 25^2.$$

So

$$49 + b^2 = 625 \quad \Rightarrow \quad b^2 = 625 - 49 = 576,$$

hence

$$b = \sqrt{576} = 24.$$

The perimeter is

$$2(7 + 24) = 2 \cdot 31 = 62.$$

6. How many distinct ways can the letters in the word "LEVEL" be arranged?

(20 pts) 6. 30

Solution: The word LEVEL has 5 letters total. The letters are:

L, E, V, E, L.

We have:

L appears 2 times, E appears 2 times, V appears 1 time.

The total number of distinct permutations of a multiset is

$$\frac{5!}{2!\,2!} = \frac{120}{4} = 30.$$

7. A school has 5 math teams and each team has 4 students. How many students must you choose to guarantee that at least two are from the same team?

(20 pts) 7. 6

Solution: There are 5 teams, each with 4 students.

To avoid having two students from the same team, you could choose at most one student from each of the 5 teams, for a total of 5 students.

As soon as you choose *one more* student (the 6th student), by the pigeonhole principle at least two of your chosen students must come from the same team.

Therefore, you must choose 6 students to guarantee that at least two are from the same team.

8. If the cost of a bat and a baseball combined is \$1.10 and the ball costs \$1.00 less than the bat, how much is the ball?

(20 pts) 8. \$0.05

Solution: Let x denote the cost of the bat and y denote the cost of the ball. We know that $x + y = \$1.10$ and $x = y + \$1.00$. Combining the second equation with the first, we produce

$$\begin{aligned} x + y &= \$1.10 \\ (y + \$1.00) + y &= \$1.10 \\ 2y + \$1.00 &= \$1.10 \\ 2y &= \$0.10 \\ y &= \$0.05. \end{aligned}$$

Therefore, the ball costs \$0.05.

9. Solve for x: $\sqrt{x+9} - \sqrt{x-7} = 2$.

(20 pts) 9. $x = 16$

Solution: Solve

$$\sqrt{x+9} - \sqrt{x-7} = 2.$$

First note the domain: we need $x - 7 \geq 0$, so $x \geq 7$.

Set

$$\sqrt{x+9} = \sqrt{x-7} + 2.$$

Now square both sides:

$$x + 9 = (\sqrt{x-7} + 2)^2 = (x - 7) + 4\sqrt{x-7} + 4.$$

Simplify the right-hand side:

$$x + 9 = x - 3 + 4\sqrt{x-7}.$$

Subtract x from both sides:

$$9 = -3 + 4\sqrt{x-7}.$$

Add 3 to both sides:

$$12 = 4\sqrt{x-7}.$$

Divide by 4:

$$3 = \sqrt{x-7}.$$

Square again:

$$9 = x - 7 \quad \Rightarrow \quad x = 16.$$

Check in the original equation:

$$\sqrt{16+9} - \sqrt{16-7} = \sqrt{25} - \sqrt{9} = 5 - 3 = 2,$$

which works.

10. In how many ways can 6 students sit in a row if two particular students insist on sitting next to each other?

(20 pts) 10. 240

Solution: Let the two particular students be A and B. They must sit next to each other.

Think of A and B as a single "block." Then we have this block plus the other 4 students, for a total of 5 objects to arrange.

The number of ways to arrange 5 distinct objects in a row is 5!.

Inside the block, the order of A and B can be either AB or BA, so there are 2! ways to arrange them internally.

Therefore, the total number of seatings with A and B together is

$$5! \cdot 2! = 120 \cdot 2 = 240.$$

Grades 11 & 12 Tests

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 1
Grades 11/12

School ______________

Team Name ______________

Calculators are allowed.

Student Name ______________

1. Find all real solutions to the equation $2^{x+1} + 2^{1-x} = 5$.

(2 pts) 1. ________

2. The expression $\dfrac{a^2 - b^2}{a - b}$ simplifies to $a+b$ for all $a \neq b$. If $a = 3+\sqrt{5}$ and $b = 3-\sqrt{5}$, find the exact value of $\dfrac{a^3 - b^3}{a - b}$.

(3 pts) 2. ________

3. A rectangle has a diagonal of length 10 cm and one side that is 2 cm longer than the other. Find the dimensions of the rectangle.

(3 pts) 3. ________

4. A surveyor stands on level ground and observes the top of a building. The line of sight to the top of the building makes an angle of elevation of 35°. The surveyor then walks 50 meters directly toward the building, where the angle of elevation increases to 50°. If the surveyor's eyes are 1.6 meters above the ground, find the horizontal distance from the first observation point to the building to the nearest meter.

(3 pts) 4. ________

5. Consider the three lines

$$L_1 : 2x + 3y = 12, \qquad L_2 : y = x - 1, \qquad L_3 : x = 2.$$

Compute the area of the triangle formed by these intersections (three decimal places).

(3 pts) 5. ________

6. A student council has 12 members: 5 seniors, 4 juniors, and 3 sophomores. A committee of 4 students is selected at random. What is the probability that the committee contains exactly two seniors? (three decimal places)

(3 pts) 6. ________

7. A landscape designer plans a triangular flower garden. Two boundary edges measure 110 m and 75 m, and the angle between them after redesign will be 63°. Find the area of the garden to the nearest square meter.

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET | INDIVIDUAL TEST 2
University of North Dakota | Grades 11/12
January 12, 2026

School ______________ Team Name ______________

Calculators are **NOT** allowed. Student Name ______________

1. Let f be a function satisfying

$$f(f(n)) + f(n) = 2n + 3,$$

and $f(n)$ is a natural number for all natural numbers n. Find $f(2026)$.

(a) 2025 (b) 2026 (c) 2027 (d) 2028 (e) 2029

(2 pts) 1. ________

2. For how many integers $n \geq 0$ is $n^2 + 6n + 5$ a perfect square?

(a) 0 (b) 1 (c) 2 (d) 3 (e) infinitely many

(3 pts) 2. ________

3. Let $P(x)$ be a nonzero polynomial satisfying

$$(x^3 + 3x^2 + 3x + 2)P(x - 1) = (x^3 - 3x^2 + 3x - 2)P(x)$$

for all real numbers x. Which is the degree of $P(x)$?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

(3 pts) 3. ________

4. Two triangles have the same perimeter. The first has side ratios $3 : 4 : 5$ and the second has ratios $7 : 24 : 25$. What is the ratio of their areas?

(a) 5 : 7 (b) 7 : 5 (c) 14 : 9 (d) 35 : 32 (e) 5 : 4

(3 pts) 4. ________

5. Triangle ABC has area 1. A point M moves along side BC. Through M, draw a line parallel to AC meeting AB at D, and a line parallel to AB meeting AC at E. The quadrilateral $ADME$ is a parallelogram. Find the maximum possible area of parallelogram $ADME$.

(a) 1 (b) 2 (c) 1/2 (d) 3/2 (e) 3

(3 pts) 5. ________

6. A teacher has 300 identical books and wants to pack them into boxes with a different number of books in each box. What is the greatest possible number of boxes?

(a) 23 (b) 24 (c) 25 (d) 26 (e) none of these

(3 pts) 6. ________

7. How many pairs of integers (x, y) satisfy the equation $\dfrac{x^2 + y^2}{x + y} = \dfrac{85}{13}$?

(a) 1 (b) 2 (c) 3 (d) 4 (e) no integer is satisfied

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 3
Grades 11/12

School _______________ Team Name _______________

Calculators are allowed. Student Name _______________

1. Find the value(s) of a such that the equation $3x^2 - 5x + a = 0$ has exactly one solution. Give an exact answer.

(2 pts) 1. _________

2. An investment account is setup with an initial investment and, after 25 years, contains one million dollars. If the amount in the account doubled every 7 years while increasing continuously, find the initial amount invested. Give your answer in dollars and cents.

(3 pts) 2. _________

3. A bag contains 3 blue, 2 green, and 5 red marbles. What's the probability that there are still green marbles in the bag after 4 marbles have been removed? Give the exact answer.

(3 pts) 3. _________

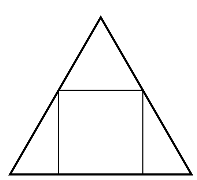
4. A group of people is randomly selected to come to a soccer field. Find the minimum number of people required to ensure that it's possible to form two teams of 11 people such that any two teammates have the same birthday (though are not necessarily the same age). Remember, February 29th is a valid birthday for those born on leap years!

(3 pts) 4. _________

5. Find the units place of the sum $1 + 2026 + 2026^2 + 2026^3 + \ldots + 2026^{2026}$

(3 pts) 5. _________

6. A square is contained in an equilateral triangle with side length 2 such that the bottom edge of the square lies along the base of the triangle and the two top vertices of the square lie on the other two edges of the triangle as shown in the figure below. Find the side length of the square. You may round your answer to two decimal places.



(3 pts) 6. _________

7. A spiral staircase climbs the outer wall of a 20 story circular tower with radius 10 meters. Given that the staircase completes one rotation every 3 stories and each story is 3 meters tall, find the (straight line) distance, in meters, between the bottom and top of the staircase. You may round your answer to two decimal places.

(3 pts) 7. _________

TOTAL POINTS _________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 4
Grades 11/12

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Student Name ______________

1. x and y are non-integers with $3x - 7y = 10$ and $xy = -1$, find the value of $9x^2 + 49y^2$.

(2 pts) 1. ________

2. In a room of 20 people, if each each person shakes each other person's hand exactly one time, how many handshakes occur?

(3 pts) 2. ________

3. Find all positive integers $n < 50$ such that $n^2 + n + 41$ is divisible by 41.

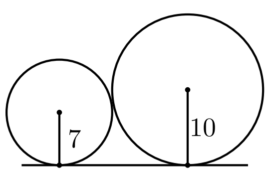
(3 pts) 3. ________

4. Find all real numbers x, y, z satisfying the system of equations.

$$x^2 - 4y + 7 = 0$$
$$y^2 - 6z + 14 = 0$$
$$z^2 - 2x - 7 = 0$$

(3 pts) 4. ________

5. The circles below are tangent to the horizontal line and tangent to each other. They have radii of 7 and 10 respectively. Find the distance between a and b.

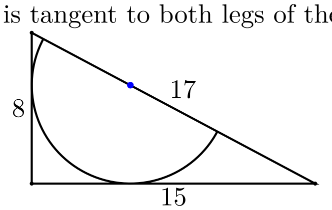


(3 pts) 5. ________

6. A standard deck of 52 playing cards is shuffled and laid out in a row. What is the probability that the four aces appear in alphabetical order from left to right? (The aces don't need to be consecutive; there can be other cards between them. We only care that when looking at the row of cards left to right, we encounter the aces in the order: Ace of Clubs, Ace of Diamonds, Ace of Hearts, Ace of Spades)

(3 pts) 6. ________

7. Find the radius of the semicircle enclosed within the right triangle. The semicircle is tangent to both legs of the right triangle.



(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET — TEAM TEST 1
University of North Dakota — Grades 11/12
January 12, 2026

School ______________ Team Name ______________

Calculators are allowed.

1. A rectangular piece of plywood is 3.7 feet wide and 7.6 feet long. Find the area of the piece of plywood. Round your answer to one decimal place. Express your answer in square feet.

(20 pts) 1. _________

2. The tires of a truck are exactly 4 feet in diameter. As the truck travels, the wheels of the truck rotate at the rate of 5 revolutions per second. How fast is the truck traveling? Express your answer in feet per second, and round your answer to one decimal place.

(20 pts) 2. _________

3. A scientist predicts that t hours from now, a petri dish will contain $39 \cdot 2^{3t}$ bacteria (39 times 2^{3t} bacteria). After how many hours will there be 497 bacteria in the petri dish? Round your answer to one decimal place.

(20 pts) 3. _________

4. Amanda is 6 inches taller than Tom. The sum of the heights of Amanda and Tom is 112 inches. Find Tom's height, in inches.

(20 pts) 4. _________

5. A large jar contains 4 yellow balls, 7 blue balls, and 6 red balls. Mrs. Smith randomly selects one ball from the jar. Then, *without* putting this ball back into the jar, she randomly selects a second ball from the jar. What is the probability that the two selected balls are both yellow? Express your answer as a percentage, and round it to one decimal place.

(20 pts) 5. _________

6. Consider the polynomial $x^3 - 4x^2 - 11x + 30$. Factor this polynomial completely.

   Hint: Let $f(x) = x^3 - 4x^2 - 11x + 30$. Let c be any real number. Then $x - c$ is a factor of $x^3 - 4x^2 - 11x + 30$ if and only if $f(c) = 0$. You may wish to experiment with some obvious choices of c. Try to find a real number c such that $f(c) = 0$. If you find such a c, this may help you find a correct solution to this problem.

(20 pts) 6. _________

7. The front row of a movie theater consists of five seats. These seats are permanently attached to the floor and cannot be rearranged. A group of five students arrives at the theater, and these students would like to sit together in the front row of the theater. Three of the students are boys, and two are girls. The students all agree that the two girls must *not* sit in adjacent seats. In how many different ways can the five students be arranged in the five seats of the front row?

(20 pts) 7. _________

8. In this problem, we consider the rightmost two digits of various positive integers. For example, the rightmost two digits of 539 are 39. The rightmost two digits of 255201 are 01.

   What are the rightmost two digits of 7^{998795}?

(20 pts) 8. _________

9. Consider a group of five towns, no three of which lie along a single straight line. We wish to construct a railway network connecting these five towns. The network must consist of four straight tracks. Each track must start at one town and end at another town. It must be possible for a train to travel from any of the five towns to any of the other four towns by using the tracks of the network. Two tracks may cross at a point P, where P does not lie in any town; if this happens, however, a train may *not* move from one track to the other at P. But if any two tracks go to the same town, a train *may* move from one of these tracks to the other at that town. How many different railway networks are possible?

(20 pts) 9. _________

10. Consider n lines in the plane, where $n > 0$. Suppose that no two of these lines are parallel and no two of them are the same line. Suppose that no single point in the plane lies on more than two different lines. (So you never have more than two lines intersecting at the same point.) The n lines divide the plane into different regions. The number of such regions depends on n. Into how many regions do the n lines divide the plane?

(20 pts) 10. _________

TOTAL POINTS _________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

TEAM TEST 2
Grades 11/12

School _______________ Team Name _______________

Calculators are **NOT** allowed.

1. Simplify $\log_{10} 6250 + \log_{10} 16$.

(20 pts) 1. _________

2. How many numbers from 1 to 120 (including 1 and 120) are divisible by 4 or 5?

(20 pts) 2. _________

3. Find all positive values for a diameter of a circle for which the area of the circle is equal numerically to half of its circumference.

(20 pts) 3. _________

4. Simplify

$$\frac{1}{\frac{1}{5} - 0.\overline{18}}$$

(20 pts) 4. _________

5. Solve $2\sqrt{x} = \sqrt{9 + \sqrt{72}} + \sqrt{9 - \sqrt{72}}$.

(20 pts) 5. _________

6. How many sides does a dodecagon have?

(20 pts) 6. _________

7. Find the area of a square with perimeter 56.

(20 pts) 7. _________

8. In degrees, what acute angle does the hour and minute hands make at 3:30?

(20 pts) 8. _________

9. A set of 5 positive integers has a median of 17 and mean of 13. What is the largest possible value in the set?

(20 pts) 9. _________

10. Find the number of ways that a red and a blue die can be rolled so that their product is a multiple of 6. Assume each die is standard with 6-sides.

(20 pts) 10. _________

TOTAL POINTS _________

Grades 11 & 12 Keys and Solutions

UND MATHEMATICS TRACK MEET | INDIVIDUAL TEST 1
University of North Dakota | Grades 11/12
January 12, 2026

School ______________ | Team Name ______________

Calculators are allowed. | **Key** | Student Name ______________

1. Find all real solutions to the equation $2^{x+1} + 2^{1-x} = 5$.

(2 pts) 1. $-1, 1$

2. The expression $\dfrac{a^2 - b^2}{a - b}$ simplifies to $a+b$ for all $a \neq b$. If $a = 3+\sqrt{5}$ and $b = 3-\sqrt{5}$, find the exact value of $\dfrac{a^3 - b^3}{a - b}$.

(3 pts) 2. 32

3. A rectangle has a diagonal of length 10 cm and one side that is 2 cm longer than the other. Find the dimensions of the rectangle.

(3 pts) 3. 6cm×8cm

4. A surveyor stands on level ground and observes the top of a building. The line of sight to the top of the building makes an angle of elevation of 35°. The surveyor then walks 50 meters directly toward the building, where the angle of elevation increases to 50°. If the surveyor's eyes are 1.6 meters above the ground, find the horizontal distance from the first observation point to the building to the nearest meter.

(3 pts) 4. 121 m

5. Consider the three lines

$$L_1 : 2x + 3y = 12, \qquad L_2 : y = x - 1, \qquad L_3 : x = 2.$$

Compute the area of the triangle formed by these intersections (three decimal places).

(3 pts) 5. 0.833

6. A student council has 12 members: 5 seniors, 4 juniors, and 3 sophomores. A committee of 4 students is selected at random. What is the probability that the committee contains exactly two seniors? (three decimal places)

(3 pts) 6. 0.424

7. A landscape designer plans a triangular flower garden. Two boundary edges measure 110 m and 75 m, and the angle between them after redesign will be 63°. Find the area of the garden to the nearest square meter.

(3 pts) 7. 3675 m^2

UND MATHEMATICS TRACK MEET — INDIVIDUAL TEST 1
University of North Dakota — Grades 11/12
January 12, 2026

School _______________ Team Name _______________

Calculators are allowed. **Solutions** Student Name _______________

1. Find all real solutions to the equation $2^{x+1} + 2^{1-x} = 5$.

(2 pts) 1. $-1, 1$

Solution: Multiply both sides by 2^x:

$$2^{2x+1} + 2 = 5 \cdot 2^x.$$

Let $t = 2^x$, $t > 0$. Then $2t^2 - 5t + 2 = 0 \Rightarrow (2t-1)(t-2) = 0$. So $t = \frac{1}{2}$ or 2. Hence $x = -1$ or 1.

$$\boxed{x = -1,\ 1.}$$

2. The expression $\dfrac{a^2 - b^2}{a - b}$ simplifies to $a+b$ for all $a \neq b$. If $a = 3+\sqrt{5}$ and $b = 3-\sqrt{5}$, find the exact value of $\dfrac{a^3 - b^3}{a - b}$.

(3 pts) 2. 32

Solution: We know $\dfrac{a^3 - b^3}{a - b} = a^2 + ab + b^2$. Compute:

$ab = (3+\sqrt{5})(3-\sqrt{5}) = 9-5 = 4, \quad a^2+b^2 = (3+\sqrt{5})^2+(3-\sqrt{5})^2 = 18+2\times 5 = 28.$

Hence $\boxed{a^2 + ab + b^2 = 28 + 4 = 32.}$

3. A rectangle has a diagonal of length 10 cm and one side that is 2 cm longer than the other. Find the dimensions of the rectangle.

(3 pts) 3. 6cm×8cm

Solution: Let shorter side x. Then longer side $x+2$. By Pythagoras: $x^2+(x+2)^2 = 10^2 \Rightarrow 2x^2+4x+4 = 100$. $2x^2+4x-96 = 0 \Rightarrow x^2+2x-48 = 0$. $x = 6cm$ (positive root). Dimensions: 6 cm,8 cm.

4. A surveyor stands on level ground and observes the top of a building. The line of sight to the top of the building makes an angle of elevation of 35°. The surveyor then walks 50 meters directly toward the building, where the angle of elevation increases to 50°. If the surveyor's eyes are 1.6 meters above the ground, find the horizontal distance from the first observation point to the building to the nearest meter.

(3 pts) 4. $121\ m$

Solution: Let H = height of the building and x = initial distance.

$$\tan(35^\circ) = \frac{H - 1.6}{x}, \qquad \tan(50^\circ) = \frac{H - 1.6}{x - 50}.$$

From the first, $H = x\tan(35^\circ) + 1.6$. Substituting in the second:

$$x = \frac{50\tan(50^\circ)}{\tan(50^\circ) - \tan(35^\circ)} \approx 121.2 \Rightarrow \boxed{x \approx 121 \text{ m}}.$$

5. Consider the three lines

$$L_1 : 2x + 3y = 12, \qquad L_2 : y = x - 1, \qquad L_3 : x = 2.$$

Compute the area of the triangle formed by these intersections (three decimal places).

(3 pts) 5. 0.833

Solution:

$$L_2 \cap L_3 : \ (2, 1), \qquad L_1 \cap L_3 : \ \left(2, \tfrac{8}{3}\right), \qquad L_1 \cap L_2 : \ (3, 2).$$

Using the shoelace formula for $A(2, 1), B(2, \frac{8}{3}), C(3, 2)$:

$$\text{Area} = \tfrac{1}{2} \left(2\left(\tfrac{8}{3} - 2\right) + 2(2 - 1) + 3\left(1 - \tfrac{8}{3}\right)\right) = \frac{5}{6}.$$

Decimal: $\boxed{0.833}$

6. A student council has 12 members: 5 seniors, 4 juniors, and 3 sophomores. A committee of 4 students is selected at random. What is the probability that the committee contains exactly two seniors? (three decimal places)

(3 pts) 6. 0.424

Solution:

$$P(\text{exactly 2 seniors}) = \frac{\binom{5}{2}\binom{7}{2}}{\binom{12}{4}} = \frac{10 \times 21}{495} = \boxed{0.424}.$$

7. A landscape designer plans a triangular flower garden. Two boundary edges measure 110 m and 75 m, and the angle between them after redesign will be 63°. Find the area of the garden to the nearest square meter.

(3 pts) 7. 3675 m^2

Solution: Using the formula for the area of a triangle:

$$A = \tfrac{1}{2} ab \sin(C),$$

with $a = 110$, $b = 75$, $C = 63°$:

$$A = \tfrac{1}{2}(110)(75)\sin(63°) \approx 3675.$$

$$\boxed{A \approx 3675 \text{ m}^2.}$$

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 2
Grades 11/12

School ______________ Team Name ______________

Calculators are **NOT** allowed. **Key** Student Name ______________

1. Let f be a function satisfying

$$f(f(n)) + f(n) = 2n + 3,$$

and $f(n)$ is a natural number for all natural numbers n. Find $f(2026)$.

(a) 2025 (b) 2026 (c) 2027 (d) 2028 (e) 2029

(2 pts) 1. (c)

2. For how many integers $n \geq 0$ is $n^2 + 6n + 5$ a perfect square?

(a) 0 (b) 1 (c) 2 (d) 3 (e) infinitely many

(3 pts) 2. (a)

3. Let $P(x)$ be a nonzero polynomial satisfying

$$(x^3 + 3x^2 + 3x + 2)P(x - 1) = (x^3 - 3x^2 + 3x - 2)P(x)$$

for all real numbers x. Which is the degree of $P(x)$?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

(3 pts) 3. (e)

4. Two triangles have the same perimeter. The first has side ratios $3 : 4 : 5$ and the second has ratios $7 : 24 : 25$. What is the ratio of their areas?

(a) $5 : 7$ (b) $7 : 5$ (c) $14 : 9$ (d) $35 : 32$ (e) $5 : 4$

(3 pts) 4. (c)

5. Triangle ABC has area 1. A point M moves along side BC. Through M, draw a line parallel to AC meeting AB at D, and a line parallel to AB meeting AC at E. The quadrilateral $ADME$ is a parallelogram. Find the maximum possible area of parallelogram $ADME$.

(a) 1 (b) 2 (c) 1/2 (d) 3/2 (e) 3

(3 pts) 5. (c)

6. A teacher has 300 identical books and wants to pack them into boxes with a different number of books in each box. What is the greatest possible number of boxes?

(a) 23 (b) 24 (c) 25 (d) 26 (e) none of these

(3 pts) 6. (b)

7. How many pairs of integers (x, y) satisfy the equation $\dfrac{x^2 + y^2}{x + y} = \dfrac{85}{13}$?

(a) 1 (b) 2 (c) 3 (d) 4 (e) no integer is satisfied

(3 pts) 7. (b)

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 2
Grades 11/12

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Solutions

Student Name ______________

1. Let f be a function satisfying

$$f(f(n)) + f(n) = 2n + 3,$$

and $f(n)$ is a natural number for all natural numbers n. Find $f(2026)$.

(a) 2025 (b) 2026 (c) 2027 (d) 2028 (e) 2029

(2 pts) 1. ___(c)___

Solution: Since f is an increasing function, we must have $f(n) > n$ for all n. Indeed, suppose on the contrary that $f(n) \leq n$ for some n, we have

$$f(f(n)) + f(n) \leq f(n) + n \leq n + n = 2n > 2n + 3,$$

which is a contradiction. Putting $g(n) = f(n) - n$, we must have $g(n) > 0$ for all n, which deduces that

$$f(f(n)) = f(n) + g(f(n)) = g(n) + n + g(n + g(n)).$$

Combining this with the assumption, we have

$$2n + 3 = f(f(n)) + f(n) = 2(g(n) + n) + g(n + g(n)).$$

In other words,

$$2g(n) + g(n + g(n)) = 3,$$

which implies that $0 < g(n) \leq 3/2$ for all n. Since $g(n)$ is a natural number, we must have $g(n) = 1$ for all n. This means that $f(n) = n + 1$ for all n. So $f(2026) = 2027$.

2. For how many integers $n \geq 0$ is $n^2 + 6n + 5$ a perfect square?

(a) 0 (b) 1 (c) 2 (d) 3 (e) infinitely many

(3 pts) 2. ___(a)___

Solution: We want $n \geq 0$ such that $n^2 + 6n + 5$ is a perfect square. Rewrite

$$n^2 + 6n + 5 = (n + 3)^2 - 4.$$

Let $(n + 3)^2 - 4 = m^2$, so

$$(n + 3)^2 - m^2 = 4 \qquad \Longrightarrow \qquad (n + 3 - m)(n + 3 + m) = 4.$$

The integer factor pairs of 4 are $(1, 4)$, $(2, 2)$, $(-1, -4)$, $(-2, -2)$. Solving n from $a = n + 3 - m$, $b = n + 3 + m$, each gives either a noninteger n or $n < 0$. Thus no $n \geq 0$ works. Therefore the answer is 0.

3. Let $P(x)$ be a nonzero polynomial satisfying

$$(x^3 + 3x^2 + 3x + 2)P(x - 1) = (x^3 - 3x^2 + 3x - 2)P(x)$$

for all real numbers x. Which is the degree of $P(x)$?

(a) 2 (b) 3 (c) 4 (d) 5 (e) 6

(3 pts) 3. (e)

Solution: We can rewrite the above equation as

$$(x+2)(x^2+x+1)P(x-1) = (x-2)(x^2-x+1)P(x). \qquad (*)$$

Choose $x=2$, we deduce that $P(1)=0$. Choose $x=-2$, we deduce that $P(-2)=0$. Choose $x=-1$, we deduce from the above equation and $P(-2)=0$ that $P(-1)=0$. Choose $x=1$, we deduce from the above equation and $P(1)=0$ that $P(0)=0$. Therefore, we can find a polynomial $G(x)$ such that

$$P(x) = x(x-1)(x+1)(x+2)G(x),$$

which implies from $(*)$ that

$$(x^2+x+1)G(x-1) = (x^2-x+1)G(x),$$

or

$$\frac{G(x-1)}{(x-1)^2+(x-1)+1} = \frac{G(x)}{x^2+x+1},$$

which implies that $R(x-1)=R(x)$ for all x, where $R(x)=G(x)/(x^2+x+1)$. This means that $R(x)$ must be a constant C. Thus, $G(x)=C(x^2+x+1)$, which implies that

$$P(x) = Cx(x-1)(x+1)(x+2)(x^2+x+1).$$

Hence, the degree of $P(x)$ is 6.

4. Two triangles have the same perimeter. The first has side ratios $3:4:5$ and the second has ratios $7:24:25$. What is the ratio of their areas?

(a) $5:7$ (b) $7:5$ (c) $14:9$ (d) $35:32$ (e) $5:4$

(3 pts) 4. (c)

Solution: Let the similar triangles have scale factors s and t, so their sides are $3s, 4s, 5s$ and $7t, 24t, 25t$. Equal perimeters give

$$12s = 56t \quad \Longrightarrow \quad s = \frac{14}{3}t.$$

Both triangles are right triangles, so their areas are

$$A_1 = \frac{1}{2}(3s)(4s) = 6s^2, \qquad A_2 = \frac{1}{2}(7t)(24t) = 84t^2.$$

Thus

$$\frac{A_1}{A_2} = \frac{6s^2}{84t^2} = \frac{s^2}{14t^2} = \frac{(14/3)^2}{14} = \frac{196}{126} = \frac{14}{9}.$$

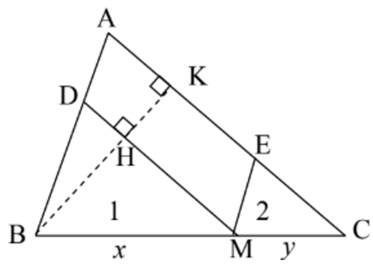
Therefore, the ratio of their areas is $14:9$.

5. Triangle ABC has area 1. A point M moves along side BC. Through M, draw a line parallel to AC meeting AB at D, and a line parallel to AB meeting AC at E. The quadrilateral $ADME$ is a parallelogram. Find the maximum possible area of parallelogram $ADME$.

(a) 1 (b) 2 (c) 1/2 (d) 3/2 (e) 3

(3 pts) 5. (c)

Solution:



Maximizing S_{ADME} is equivalent to maximizing the ratio

$$\frac{S_{ADME}}{S_{ABC}}.$$

Draw $BK \perp AC$ intersecting MD at H. Then

$$S_{ADME} = MD \cdot HK, \qquad S_{ABC} = \frac{1}{2} AC \cdot BK,$$

so

$$\frac{S_{ADME}}{S_{ABC}} = 2 \cdot \frac{MD}{AC} \cdot \frac{HK}{BK}.$$

Let $MB = x$ and $MC = y$. Since $MD \parallel AC$, we have

$$\frac{MD}{AC} = \frac{BM}{BC} = \frac{x}{x+y}, \qquad \frac{HK}{BK} = \frac{MC}{BC} = \frac{y}{x+y}.$$

By the inequality

$$\frac{xy}{(x+y)^2} \le \frac{1}{4} \quad \Rightarrow \quad \frac{S_{ADME}}{S_{ABC}} = \frac{2xy}{(x+y)^2} \le \frac{1}{2}.$$

Equality occurs when $x = y$. Therefore,

$$\max S_{ADME} = \frac{1}{2} S_{ABC} = \frac{1}{2},$$

which happens when M is the midpoint of BC.

6. A teacher has 300 identical books and wants to pack them into boxes with a different number of books in each box. What is the greatest possible number of boxes?

(a) 23 (b) 24 (c) 25 (d) 26 (e) none of these

(3 pts) 6. (b)

Solution: Suppose the boxes have sizes $1, 2, \ldots, k$. Then the total number of books is

$$1 + 2 + \cdots + k = \frac{k(k+1)}{2}.$$

We require $\frac{k(k+1)}{2} \le 300$. This inequality is equivalent to $k^2 + k - 600 \le 0$. The positive root of the quadratic equation $k^2 + k - 600 = 0$ is 24. Thus $k \le 24$. Since

$$1 + 2 + \cdots + 24 = \frac{24 \cdot 25}{2} = 300,$$

the value $k = 24$ is achievable. For $k = 25$, we would need

$$1 + 2 + \cdots + 25 = \frac{25 \cdot 26}{2} = 325 > 300,$$

which is impossible. Therefore, the greatest possible number of boxes is 24.

7. How many pairs of integers (x, y) satisfy the equation $\dfrac{x^2+y^2}{x+y} = \dfrac{85}{13}$?

(a) 1 (b) 2 (c) 3 (d) 4 (e) no integer is satisfied

(3 pts) 7. (b)

Solution: We want integers x, y such that

$$\frac{x^2+y^2}{x+y} = \frac{85}{13}, \qquad x+y \neq 0.$$

Cross–multiply:

$$13(x^2+y^2) = 85(x+y).$$

Let $s = x+y$ and $p = xy$. Then $x^2+y^2 = s^2-2p$, so

$$13(s^2-2p) = 85s \implies p = \frac{s(13s-85)}{26}.$$

For integer solutions, we need an integer p and the discriminant

$$\Delta = s^2 - 4p$$

to be a nonnegative perfect square. Substitute p:

$$\Delta = s^2 - 4 \cdot \frac{s(13s-85)}{26} = \frac{s(170-13s)}{13}.$$

Thus Δ is an integer only if $13 \mid s$, so write $s = 13k$. Then

$$\Delta = k(170-169k).$$

For $\Delta \geq 0$ we need $k = 0$ or $k = 1$. The case $k = 0$ gives $s = 0$, which is not allowed since $x+y \neq 0$. Thus $k = 1$, giving $s = 13$ and $\Delta = 1$.

Therefore

$$x, y = \frac{13 \pm 1}{2} \in \{6, 7\}.$$

The integer solutions are $(6, 7)$ and $(7, 6)$, so the answer is 2.

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 3
Grades 11/12

School ______________ Team Name ______________

Calculators are allowed. **Key** Student Name ______________

1. Find the value(s) of a such that the equation $3x^2 - 5x + a = 0$ has exactly one solution. Give an exact answer.

(2 pts) 1. $\frac{25}{12}$

2. An investment account is setup with an initial investment and, after 25 years, contains one million dollars. If the amount in the account doubled every 7 years while increasing continuously, find the initial amount invested. Give your answer in dollars and cents.

(3 pts) 2. $84,118.76

3. A bag contains 3 blue, 2 green, and 5 red marbles. What's the probability that there are still green marbles in the bag after 4 marbles have been removed? Give the exact answer.

(3 pts) 3. $\frac{182}{210} = \frac{13}{15}$

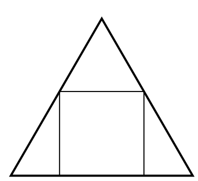
4. A group of people is randomly selected to come to a soccer field. Find the minimum number of people required to ensure that it's possible to form two teams of 11 people such that any two teammates have the same birthday (though are not necessarily the same age). Remember, February 29th is a valid birthday for those born on leap years!

(3 pts) 4. 3672

5. Find the units place of the sum $1 + 2026 + 2026^2 + 2026^3 + \ldots + 2026^{2026}$

(3 pts) 5. 7

6. A square is contained in an equilateral triangle with side length 2 such that the bottom edge of the square lies along the base of the triangle and the two top vertices of the square lie on the other two edges of the triangle as shown in the figure below. Find the side length of the square. You may round your answer to two decimal places.



(3 pts) 6. $\frac{2\sqrt{3}}{2+\sqrt{3}} \approx$ 0.93

7. A spiral staircase climbs the outer wall of a 20 story circular tower with radius 10 meters. Given that the staircase completes one rotation every 3 stories and each story is 3 meters tall, find the (straight line) distance, in meters, between the bottom and top of the staircase. You may round your answer to two decimal places.

(3 pts) 7. $\sqrt{3900} \approx$ 62.45

UND MATHEMATICS TRACK MEET | INDIVIDUAL TEST 3
University of North Dakota | Grades 11/12
January 12, 2026

School ______________ Team Name ______________

Calculators are allowed. **Solutions** Student Name ______________

1. Find the value(s) of a such that the equation $3x^2 - 5x + a = 0$ has exactly one solution. Give an exact answer.

(2 pts) 1. $\frac{25}{12}$

Solution: We need the discriminant $b^2 - 4ac = 0$ or $25 - 12a = 0$, so we want $a = 25/12$.

2. An investment account is setup with an initial investment and, after 25 years, contains one million dollars. If the amount in the account doubled every 7 years while increasing continuously, find the initial amount invested. Give your answer in dollars and cents.

(3 pts) 2. $84,118.76

Solution: If s is the number of doubling times and A is the initial amount in the account, there are $A2^s$ dollars in the account. After $s = 25/7$ doubling times, there are a million dollars, so we solve $1,000,000 = A2^{25/7}$ for A to get $A = 1,000,000 \cdot 2^{-25/7} \approx 84118.76$

3. A bag contains 3 blue, 2 green, and 5 red marbles. What's the probability that there are still green marbles in the bag after 4 marbles have been removed? Give the exact answer.

(3 pts) 3. $\frac{182}{210} = \frac{13}{15}$

Solution: We need to find the probability that we chose zero or one green marble from our four. There are $\binom{10}{4} = 210$ total outcomes for the four marbles selected. There are $\binom{8}{4} = 70$ outcomes with zero green marbles selected and $2\binom{8}{3} = 112$ outcomes with one green marble selected. Therefore, the probability is $\frac{70+112}{210} = \frac{182}{210} = \frac{13}{15}$

4. A group of people is randomly selected to come to a soccer field. Find the minimum number of people required to ensure that it's possible to form two teams of 11 people such that any two teammates have the same birthday (though are not necessarily the same age). Remember, February 29th is a valid birthday for those born on leap years!

(3 pts) 4. 3672

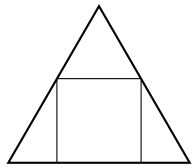
Solution: Since there are 366 possible birthdays, the worst case scenario where no team of 11 can be formed with the same birthday is 3660 people having 10 people per birthday. One more person ensures we have at least one team and we need to add 11 more after this since these 11 could all have the same birthday as the first team. So $3660 + 12 = 3672$ people are required to ensure this happens.

5. Find the units place of the sum $1 + 2026 + 2026^2 + 2026^3 + \ldots + 2026^{2026}$

(3 pts) 5. 7

Solution: Since $2026 \equiv 6 \pmod{10}$, we have $2026^2 \equiv 6^2 \equiv 6 \pmod{10}$, $2026^3 \equiv 6^3 \equiv 6^2 \equiv 6 \pmod{10}$, and so forth for every power of 2026. Therefore, $1 + 2026 + 2026^2 + \ldots + 2026^{2026} \equiv 1 + 6 + \ldots 6 \pmod{10}$. There are 2026 copies of 6 for $1 + 2026 \cdot 6 \equiv 1 + 6 \cdot 6 \equiv 7 \pmod{10}$.

6. A square is contained in an equilateral triangle with side length 2 such that the bottom edge of the square lies along the base of the triangle and the two top vertices of the square lie on the other two edges of the triangle as shown in the figure below. Find the side length of the square. You may round your answer to two decimal places.



(3 pts) 6. $\frac{2\sqrt{3}}{2+\sqrt{3}} \approx 0.93$

Solution: Let s be the square side length. Since the top of the square is parallel to the base of the triangle, it cuts off a triangle on top of the square similar to the whole triangle and hence this upper triangle must also be equilateral. This upper equilateral triangle has side lengths s and so has height $\sqrt{3}s/2$, meaning the height $\sqrt{3}$ of the large triangle equals $s + \frac{\sqrt{3}s}{2}$ allowing us to solve for s as $s = \frac{2\sqrt{3}}{2+\sqrt{3}}$.

7. A spiral staircase climbs the outer wall of a 20 story circular tower with radius 10 meters. Given that the staircase completes one rotation every 3 stories and each story is 3 meters tall, find the (straight line) distance, in meters, between the bottom and top of the staircase. You may round your answer to two decimal places.

(3 pts) 7. $\sqrt{3900} \approx 62.45$

Solution: We model horizontal position on the staircase with $x^2 + y^2 = 100$ with the bottom of the staircase at $(10, 0)$. At the top, we are $2/3$ of the way through a rotation, meaning at angle $4\pi/3$ at position $(10\cos(4\pi/3), 10\sin(4\pi/3)) = (-5, -5\sqrt{3})$. Assuming the staircase starts at $z = 0$, the initial and final positions for the staircase are $(10, 0, 0)$ and $(-5, -5\sqrt{3}, 60)$ which are separated by distance $\sqrt{(10-(-5))^2 + (0-(-5\sqrt{3}))^2 + (60-0)^2} = \sqrt{3900} \approx 62.45$

UND MATHEMATICS TRACK MEET | INDIVIDUAL TEST 4
University of North Dakota | Grades 11/12
January 12, 2026

School _______________ Team Name _______________

Calculators are **NOT** allowed. **Key** Student Name _______________

1. x and y are non-integers with $3x - 7y = 10$ and $xy = -1$, find the value of $9x^2 + 49y^2$.

(2 pts) 1. 58

2. In a room of 20 people, if each each person shakes each other person's hand exactly one time, how many handshakes occur?

(3 pts) 2. 190

3. Find all positive integers $n < 50$ such that $n^2 + n + 41$ is divisible by 41.

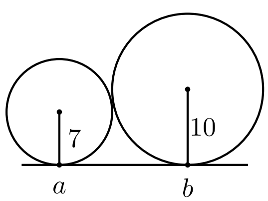
(3 pts) 3. $40, 41$

4. Find all real numbers x, y, z satisfying the system of equations.

$$\begin{aligned} x^2 - 4y + 7 &= 0 \\ y^2 - 6z + 14 &= 0 \\ z^2 - 2x - 7 &= 0 \end{aligned}$$

(3 pts) 4. $(1, 2, 3)$

5. The circles below are tangent to the horizontal line and tangent to each other. They have radii of 7 and 10 respectively. Find the distance between a and b.

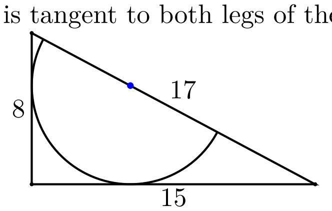


(3 pts) 5. $\sqrt{280}$

6. A standard deck of 52 playing cards is shuffled and laid out in a row. What is the probability that the four aces appear in alphabetical order from left to right? (The aces don't need to be consecutive; there can be other cards between them. We only care that when looking at the row of cards left to right, we encounter the aces in the order: Ace of Clubs, Ace of Diamonds, Ace of Hearts, Ace of Spades)

(3 pts) 6. $1/24$

7. Find the radius of the semicircle enclosed within the right triangle. The semicircle is tangent to both legs of the right triangle.



(3 pts) 7. $120/23$

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

INDIVIDUAL TEST 4
Grades 11/12

School _______________

Team Name _______________

Calculators are **NOT** allowed.

Solutions

Student Name _______________

1. x and y are non-integers with $3x-7y=10$ and $xy=-1$, find the value of $9x^2+49y^2$.

(2 pts) 1. 58

Solution: $3x-7y=10$ and $xy=-1$ means that $(\frac{7y+10}{3})y=-1$.

Simplifying to a standard form quadratic, we get $7y^2+10y+3=0$ which factors as $(7y+3)(y+1)=0$. This yields solutions $y=-\frac{3}{7}$ and $y=-1$. We can eliminate the integer solution.
That means $x=\frac{7}{3}$ and $y=-\frac{3}{7}$. Substituting into $9x^2+49y^2$ results in $9(\frac{49}{9})+49(\frac{9}{49})=58$.

2. In a room of 20 people, if each each person shakes each other person's hand exactly one time, how many handshakes occur?

(3 pts) 2. 190

Solution: $19+18+17+\cdots+2+1=\frac{19(20)}{2}=190$.

3. Find all positive integers $n<50$ such that n^2+n+41 is divisible by 41.

(3 pts) 3. $40, 41$

Solution: We need n^2+n to be divisible by 41. $n^2+n=n(n+1)$. Since 41 is prime, only $n=40$ and $n=41$ are solutions.

4. Find all real numbers x, y, z satisfying the system of equations.

$$\begin{aligned} x^2-4y+7&=0\\ y^2-6z+14&=0\\ z^2-2x-7&=0 \end{aligned}$$

(3 pts) 4. $(1,2,3)$

Solution: Adding all three equations together gives us

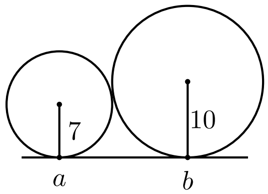
$x^2-2x+y^2-4y+z^2-6z+14=0$.

Completing the square with each quadratic leads to

$(x-1)^2+(y-2)^2+(z-3)^2=0$.

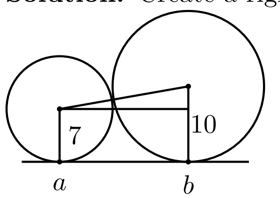
As each term is a perfect square, the only solution is $(1,2,3)$.

5. The circles below are tangent to the horizontal line and tangent to each other. They have radii of 7 and 10 respectively. Find the distance between a and b.



(3 pts) 5. $\sqrt{280}$

Solution: Create a right triangle in the following way:



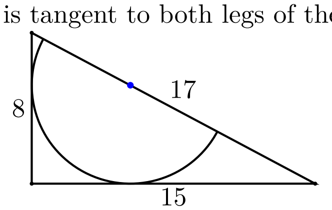
The horizontal leg of the triangle is what we need to find. The hypotenuse is 17, the vertical leg is 3. By the Pythagorean Theorem, our solution is $\sqrt{289-9} = \sqrt{280}$.

6. A standard deck of 52 playing cards is shuffled and laid out in a row. What is the probability that the four aces appear in alphabetical order from left to right? (The aces don't need to be consecutive; there can be other cards between them. We only care that when looking at the row of cards left to right, we encounter the aces in the order: Ace of Clubs, Ace of Diamonds, Ace of Hearts, Ace of Spades)

(3 pts) 6. 1/24

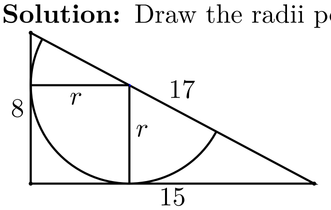
Solution: We are looking for one arrangement of the aces out of 4! possible arrangements. So, the probability is $\frac{1}{4!} = \frac{1}{24}$

7. Find the radius of the semicircle enclosed within the right triangle. The semicircle is tangent to both legs of the right triangle.



(3 pts) 7. 120/23

Solution: Draw the radii perpendicular to the legs of the triangle.



The area of the 8-15-17 triangle can be split into the area of the two smaller right triangles and the square.
$0.5(8)(15) = 0.5(8-r)r + r^2 + 0.5(15-r)r$
This simplifies to $60 = 4r - 0.5r^2 + r^2 + 7.5r - 0.5r^2$
$60 = 11.5r$ or $60 = \frac{23}{2}r$, that means $r = \frac{60}{11.5}$ or $\frac{120}{23}$

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

TEAM TEST 1
Grades 11/12

School _______________ Team Name _______________

Calculators are allowed.

Key

1. A rectangular piece of plywood is 3.7 feet wide and 7.6 feet long. Find the area of the piece of plywood. Round your answer to one decimal place. Express your answer in square feet.

(20 pts) 1. 28.1 ft^2

2. The tires of a truck are exactly 4 feet in diameter. As the truck travels, the wheels of the truck rotate at the rate of 5 revolutions per second. How fast is the truck traveling? Express your answer in feet per second, and round your answer to one decimal place.

(20 pts) 2. 62.8 ft/s

3. A scientist predicts that t hours from now, a petri dish will contain $39 \cdot 2^{3t}$ bacteria (39 times 2^{3t} bacteria). After how many hours will there be 497 bacteria in the petri dish? Round your answer to one decimal place.

(20 pts) 3. 1.2 hrs

4. Amanda is 6 inches taller than Tom. The sum of the heights of Amanda and Tom is 112 inches. Find Tom's height, in inches.

(20 pts) 4. 53 in

5. A large jar contains 4 yellow balls, 7 blue balls, and 6 red balls. Mrs. Smith randomly selects one ball from the jar. Then, *without* putting this ball back into the jar, she randomly selects a second ball from the jar. What is the probability that the two selected balls are both yellow? Express your answer as a percentage, and round it to one decimal place.

(20 pts) 5. 4.4%

6. Consider the polynomial $x^3 - 4x^2 - 11x + 30$. Factor this polynomial completely.

   Hint: Let $f(x) = x^3 - 4x^2 - 11x + 30$. Let c be any real number. Then $x - c$ is a factor of $x^3 - 4x^2 - 11x + 30$ if and only if $f(c) = 0$. You may wish to experiment with some obvious choices of c. Try to find a real number c such that $f(c) = 0$. If you find such a c, this may help you find a correct solution to this problem.

(20 pts) 6. $(x-2)(x-5)(x+3)$

7. The front row of a movie theater consists of five seats. These seats are permanently attached to the floor and cannot be rearranged. A group of five students arrives at the theater, and these students would like to sit together in the front row of the theater. Three of the students are boys, and two are girls. The students all agree that the two girls must *not* sit in adjacent seats. In how many different ways can the five students be arranged in the five seats of the front row?

(20 pts) 7. 72 ways

8. In this problem, we consider the rightmost two digits of various positive integers. For example, the rightmost two digits of 539 are 39. The rightmost two digits of 255201 are 01.

   What are the rightmost two digits of 7^{998795}?

(20 pts) 8. 43

9. Consider a group of five towns, no three of which lie along a single straight line. We wish to construct a railway network connecting these five towns. The network must consist of four straight tracks. Each track must start at one town and end at another town. It must be possible for a train to travel from any of the five towns to any of the other four towns by using the tracks of the network. Two tracks may cross at a point P, where P does not lie in any town; if this happens, however, a train may *not* move from one track to the other at P. But if any two tracks go to the same town, a train *may* move from one of these tracks to the other at that town. How many different railway networks are possible?

(20 pts) 9. 125 networks

10. Consider n lines in the plane, where $n > 0$. Suppose that no two of these lines are parallel and no two of them are the same line. Suppose that no single point in the plane lies on more than two different lines. (So you never have more than two lines intersecting at the same point.) The n lines divide the plane into different regions. The number of such regions depends on n. Into how many regions do the n lines divide the plane?

(20 pts) 10. $\frac{n^2+n+2}{2}$
or $2+(2+3+4+\cdots+n)$
or $1+(1+2+3+\cdots+n)$
or $1+\frac{n(n+1)}{2}$

UND MATHEMATICS TRACK MEET — TEAM TEST 1

University of North Dakota — Grades 11/12

January 12, 2026

School ______________ Team Name ______________

Calculators are allowed.

Solutions

1. A rectangular piece of plywood is 3.7 feet wide and 7.6 feet long. Find the area of the piece of plywood. Round your answer to one decimal place. Express your answer in square feet.

(20 pts) 1. 28.1 ft^2

This is $3.7 \cdot 7.6 = 28.12 \approx 28.1$ square feet.

2. The tires of a truck are exactly 4 feet in diameter. As the truck travels, the wheels of the truck rotate at the rate of 5 revolutions per second. How fast is the truck traveling? Express your answer in feet per second, and round your answer to one decimal place.

(20 pts) 2. 62.8 ft/s

We have

$$\frac{5\text{rev}}{1\text{sec}} \times \frac{4\pi\text{feet}}{1\text{rev}} = 20\pi\frac{\text{feet}}{\text{sec}} \approx 62.8\frac{\text{feet}}{\text{sec}}$$

3. A scientist predicts that t hours from now, a petri dish will contain $39 \cdot 2^{3t}$ bacteria (39 times 2^{3t} bacteria). After how many hours will there be 497 bacteria in the petri dish? Round your answer to one decimal place.

(20 pts) 3. 1.2 hrs

We have $39 \cdot 2^{3t} = 497$ when $2^{3t} = 497/39$. Taking logarithms, this means $3t \log 2 = \log(497/39)$, so that $t = \frac{\log(497/39)}{3\log 2} = 1.2$ hours.

4. Amanda is 6 inches taller than Tom. The sum of the heights of Amanda and Tom is 112 inches. Find Tom's height, in inches.

(20 pts) 4. 53 in

We have $(T + 6) + T = 112$, so $T = 53$ inches.

5. A large jar contains 4 yellow balls, 7 blue balls, and 6 red balls. Mrs. Smith randomly selects one ball from the jar. Then, *without* putting this ball back into the jar, she randomly selects a second ball from the jar. What is the probability that the two selected balls are both yellow? Express your answer as a percentage, and round it to one decimal place.

(20 pts) 5. 4.4%

This is $\frac{4}{17} \cdot \frac{3}{16} \approx 0.04412 \approx 4.4\%$.

6. Consider the polynomial $x^3 - 4x^2 - 11x + 30$. Factor this polynomial completely.

Hint: Let $f(x) = x^3 - 4x^2 - 11x + 30$. Let c be any real number. Then $x - c$ is a factor of $x^3 - 4x^2 - 11x + 30$ if and only if $f(c) = 0$. You may wish to experiment with some obvious choices of c. Try to find a real number c such that $f(c) = 0$. If you find such a c, this may help you find a correct solution to this problem.

(20 pts) 6. $(x-2)(x-5)(x+3)$

Following the hint, we find that $f(2) = 0$ so that $(x - 2)$ is a factor. Using long division, $f(x) = (x - 2)(x^2 - 2x - 15) = (x - 2)(x - 5)(x + 3)$.

7. The front row of a movie theater consists of five seats. These seats are permanently attached to the floor and cannot be rearranged. A group of five students arrives at the theater, and these students would like to sit together in the front row of the theater. Three of the students are boys, and two are girls. The students all agree that the two girls must *not* sit in adjacent seats. In how many different ways can the five students be arranged in the five seats of the front row?

(20 pts) 7. 72 ways

There are $5! = 120$ ways for everyone to sit, of which $4! \cdot 2 = 48$ have the two girls together. Therefore, there are 72 ways for them not to be together.

8. In this problem, we consider the rightmost two digits of various positive integers. For example, the rightmost two digits of 539 are 39. The rightmost two digits of 255201 are 01.

What are the rightmost two digits of 7^{998795}?

(20 pts) 8. 43

Notice that $7^2 = 49$, $7^3 = 343$, and $7^4 = 2401$ has the rightmost two digits 01. Since $998795 = 4 \cdot 249698 + 3$, $7^{998795} = 7^{4 \cdot 249698 + 3} = (7^4)^{249698} 7^3$ has the rightmost two digits that are the same as $01^{249698} 343 = 343$. Therefore, the rightmost two digits are 43.

9. Consider a group of five towns, no three of which lie along a single straight line. We wish to construct a railway network connecting these five towns. The network must consist of four straight tracks. Each track must start at one town and end at another town. It must be possible for a train to travel from any of the five towns to any of the other four towns by using the tracks of the network. Two tracks may cross at a point P, where P does not lie in any town; if this happens, however, a train may *not* move from one track to the other at P. But if any two tracks go to the same town, a train *may* move from one of these tracks to the other at that town. How many different railway networks are possible?

(20 pts) 9. 125 networks

There are a few cases to consider. If each town has at most two tracks, then the network forms a path starting at one town, going through each town in turn, and ending at another town. There are 10 ways to choose two towns from the five available, and then 6 differently ordered ways to visit the three remaining towns to give a total of 60 ways. If some town has all four tracks, the network forms a star with that town at the center, and there are 5 ways to choose this center. The remaining case is that some town has three tracks, of which there are 5 ways to choose this town, and therefore some town that is not directly connected to this first town. There are 4 ways to choose this last town, and we have 3 options on which town to connect it, for a total of 60 ways to form our network. Therefore, there are $60 + 5 + 60 = 125$ possible networks.

10. Consider n lines in the plane, where $n > 0$. Suppose that no two of these lines are parallel and no two of them are the same line. Suppose that no single point in the plane lies on more than two different lines. (So you never have more than two lines intersecting at the same point.) The n lines divide the plane into different regions. The number of such regions depends on n. Into how many regions do the n lines divide the plane?

(20 pts) 10. $\frac{n^2+n+2}{2}$
or $2+(2+3+4+\cdots+n)$
or $1+(1+2+3+\cdots+n)$
or $1+\frac{n(n+1)}{2}$

If $k-1$ lines are already in the plane, then the k^{th} line will cross each of them, going through k different regions. In each region, it will increase the number of regions by one. Therefore, the k^{th} line will create k new regions. Since we start with one line making two regions, the total number of regions is therefore $2+(2+3+4+\cdots+n) = 1+(1+2+3+4+\cdots+n) = 1+\frac{n(n+1)}{2} = \frac{n^2+n+2}{2}$. (This also works when $n=1$.)

UND MATHEMATICS TRACK MEET | TEAM TEST 2
University of North Dakota | Grades 11/12
January 12, 2026

School ______________ Team Name ______________

Calculators are **NOT** allowed. **Key**

1. Simplify $\log_{10} 6250 + \log_{10} 16$.

   (20 pts) 1. 5

2. How many numbers from 1 to 120 (including 1 and 120) are divisible by 4 or 5?

   (20 pts) 2. 48

3. Find all positive values for a diameter of a circle for which the area of the circle is equal numerically to half of its circumference.

   (20 pts) 3. 2

4. Simplify

$$\frac{1}{\frac{1}{5} - 0.\overline{18}}$$

   (20 pts) 4. 55

5. Solve $2\sqrt{x} = \sqrt{9+\sqrt{72}} + \sqrt{9-\sqrt{72}}$.

   (20 pts) 5. 6

6. How many sides does a dodecagon have?

   (20 pts) 6. 12

7. Find the area of a square with perimeter 56.

   (20 pts) 7. 196

8. In degrees, what acute angle does the hour and minute hands make at 3:30?

   (20 pts) 8. 75°

9. A set of 5 positive integers has a median of 17 and mean of 13. What is the largest possible value in the set?

   (20 pts) 9. 29

10. Find the number of ways that a red and a blue die can be rolled so that their product is a multiple of 6. Assume each die is standard with 6-sides.

   (20 pts) 10. 15

UND MATHEMATICS TRACK MEET
University of North Dakota
January 12, 2026

TEAM TEST 2
Grades 11/12

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Solutions

1. Simplify $\log_{10} 6250 + \log_{10} 16$.

(20 pts) 1. 5

Solution: We have $\log_{10} 6250 + \log_{10} 16 = \log_{10}(2 \cdot 5^5) + \log_{10} 2^4 = \log_{10}(2^5 \cdot 5^5) = \log_{10} 10^5 = 5$.

2. How many numbers from 1 to 120 (including 1 and 120) are divisible by 4 or 5?

(20 pts) 2. 48

Solution: Of the numbers from 1 to 120, there are $\lfloor 120/4 \rfloor = 30$ divisible by 4, $\lfloor 120/5 \rfloor = 24$ divisible by 5, and $\lfloor 120/\text{lcm}(4,5) \rfloor = \lfloor 120/20 \rfloor = 6$ divisible by 4 and 5. By inclusion-exclusion counting, there are $30 + 24 - 6 = 48$ such numbers.

3. Find all positive values for a diameter of a circle for which the area of the circle is equal numerically to half of its circumference.

(20 pts) 3. 2

Solution: Let d be the diameter of a circle. The condition $A = \frac{1}{2}C$ for the area A and circumference C can be expressed in terms of d as $\pi\left(\frac{d}{2}\right)^2 = \frac{1}{2}(\pi d)$. Rearranging gives $\pi d^2 - 2\pi d = 0$ or equivalently $\pi d(d-2) = 0$. The only positive solution to the latter is $d = 2$.

4. Simplify

$$\frac{1}{\frac{1}{5} - 0.\overline{18}}$$

(20 pts) 4. 55

Solution:

$$\frac{1}{\frac{1}{5} - 0.\overline{18}} = \frac{1}{\frac{1}{5} - \frac{18}{99}} = \frac{1}{\frac{99}{5(99)} - \frac{5(18)}{5(99)}} = \frac{1}{\frac{9}{5(99)}} = 5(11) = 55.$$

5. Solve $2\sqrt{x} = \sqrt{9 + \sqrt{72}} + \sqrt{9 - \sqrt{72}}$.

(20 pts) 5. 6

Solution: Observe squaring $2\sqrt{x} = \sqrt{9 + \sqrt{72}} + \sqrt{9 - \sqrt{72}}$ gives

$$\begin{aligned} 4x &= \left(\sqrt{9 + \sqrt{72}} + \sqrt{9 - \sqrt{72}}\right)^2 \\ 4x &= 9 + \sqrt{72} + 2\sqrt{9 + \sqrt{72}} \cdot \sqrt{9 - \sqrt{72}} + \left(9 - \sqrt{72}\right) \\ 4x &= 9 + \sqrt{72} + 2\sqrt{9} + \left(9 - \sqrt{72}\right) \\ 4x &= 24. \end{aligned}$$

The latter has solution $x = 6$. A check verifies that $x = 6$ is a solution to the given equation.

6. How many sides does a dodecagon have?

(20 pts) 6. 12

Solution: A dodecagon has 12 sides.

7. Find the area of a square with perimeter 56.

(20 pts) 7. 196

Solution: Let s be the side length of a square. If the perimeter P of the square is 56, we have $4s = 56$, so $s = 14$. The area of the square is $A = s^2 = 14^2 = 196$.

8. In degrees, what acute angle does the hour and minute hands make at 3:30?

(20 pts) 8. 75°

Solution: At 3:30, the hour hand lies halfway between 3 and 4 hour marks on the 12-hour analog clock. There are $\frac{360°}{12} = 30°$ between each hour marks on the clock. Thus the angle at 3:30 between the minute and hour hand is $2(30°) + \frac{1}{2}(30°) = 75°$.

9. A set of 5 positive integers has a median of 17 and mean of 13. What is the largest possible value in the set?

(20 pts) 9. 29

Solution: Let $n_1 \leq n_2 \leq n_3 \leq n_4 \leq n_5$ be the five positive integers. Since the median is 17, we have $n_3 = 17$. A mean of 13 implies

$$\frac{n_1 + n_2 + 17 + n_4 + n_5}{5} = 13.$$

Rearranging gives $n_1 + n_2 + n_4 + n_5 = 48$. To maximize n_5 subject to the conditions given, take $n_1 = n_2 = 1$ and $n_4 = 17$. This gives $n_5 = 29$.

10. Find the number of ways that a red and a blue die can be rolled so that their product is a multiple of 6. Assume each die is standard with 6-sides.

(20 pts) 10. 15

Solution: Let (r, b) be a roll outcome. Then rb is a multiple of 6 if and only if $(1, 6)$, $(2, 3)$, $(2, 6)$, $(3, 2)$, $(3, 4)$, $(3, 6)$, $(4, 3)$, $(4, 6)$, $(5, 6)$, $(6, 1)$, $(6, 2)$, $(6, 3)$, $(6, 4)$, $(6, 5)$, $(6, 6)$. Thus, the number of ways is 15.