2025 Math Track Meet

University of North Dakota

January 13, 2023

Contents

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #1
Grades 7/8

School ______________

Team Name ______________

Calculators are allowed.

Student Name ______________

1. Solve for x. Express x as an integer or a fraction where the numerator and denominator have no common factors.

$$1 - \frac{3x}{2} = \frac{5}{4}$$

(2 pts) 1. ________

2. Three coins are tossed, each of which are equally likely to come up heads or tails. What is the probability that exactly 2 of the coins come up heads?

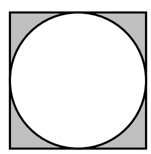
(3 pts) 2. ________

3. Reduce the following expression to a simple fraction where the numerator and denominator have no common factors.

$$\cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2}}}$$

(3 pts) 3. ________

4. A circle is inscribed in a square as shown. The square has perimeter 64 cm. How many square centimeters are in the square but not in the circle (the shaded region of the figure)? Use 3.14 for π and express your answer to the nearest hundredth of a square centimeter



(3 pts) 4. ________

5. What is the largest prime factor of 2025?

(3 pts) 5. ________

6. Four students take a quiz, and each receives a score which is a whole number. The highest score is 10, and the lowest is 2. If the mean (average) score is 8, what is the mode of the scores?

(3 pts) 6. ________

7. The degree measures of the angles of a triangle equal $2x$,$3x$, and $4x$ for some x. What is the measure (in degrees) of the greatest angle?

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #1
Grades 7/8

School ______________

Team Name ______________

Calculators are allowed.

Key

Student Name ______________

1. Solve for x. Express x as an integer or a fraction where the numerator and denominator have no common factors.

$$1 - \frac{3x}{2} = \frac{5}{4}$$

(2 pts) 1. $-\frac{1}{6}$

2. Three coins are tossed, each of which are equally likely to come up heads or tails. What is the probability that exactly 2 of the coins come up heads?

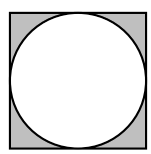
(3 pts) 2. $\frac{3}{8}$ or 0.375 or 37.5%

3. Reduce the following expression to a simple fraction where the numerator and denominator have no common factors.

$$\cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2}}}$$

(3 pts) 3. $\frac{3}{4}$

4. A circle is inscribed in a square as shown. The square has perimeter 64 cm. How many square centimeters are in the square but not in the circle (the shaded region of the figure)? Use 3.14 for π and express your answer to the nearest hundredth of a square centimeter



(3 pts) 4. 55.04

5. What is the largest prime factor of 2025?

(3 pts) 5. 5

6. Four students take a quiz, and each receives a score which is a whole number. The highest score is 10, and the lowest is 2. If the mean (average) score is 8, what is the mode of the scores?

(3 pts) 6. 10

7. The degree measures of the angles of a triangle equal $2x$,$3x$, and $4x$ for some x. What is the measure (in degrees) of the greatest angle?

(3 pts) 7. 80

FORKS AREA MATHEMATICS TRACK MEET | INDIVIDUAL TEST #1
University of North Dakota | Grades 7/8
January 15, 2025

School ______________________ Team Name ________________________

Calculators are allowed. Student Name ________________________

1. Solve for x. Express x as an integer or a fraction where the numerator and denominator have no common factors.

$$1-\frac{3x}{2}=\frac{5}{4}$$

(2pts) 1. __ $\mathbf{-1/6}$ ____

$$4-6x=5 \Rightarrow -6x=1 \Longrightarrow x=-1/6.$$

2. Three coins are tossed, each of which are equally likely to come up heads or tails. What is the probability that exactly 2 of the coins come up heads? (3pts) 2. **_3/8_or_0.375_**

There are 8 possible outcomes, all equally likely: {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}. Three (HHT, HTH, THH) have exactly 2 heads. So the probability is 3/8.

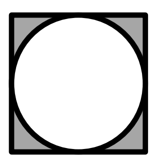
3. Reduce the following expression to a simple fraction where the numerator and denominator have no common factors.

$$\frac{1}{2-\frac{1}{2-\frac{1}{2}}}$$

(3pts) 3. ____ $\mathbf{3/4}$ _____

$$\frac{1}{2-\frac{1}{2-\frac{1}{2}}}=\frac{1}{2-\frac{1}{\frac{3}{2}}}=\frac{1}{2-\frac{2}{3}}=\frac{1}{\frac{4}{3}}=\frac{3}{4}$$

4. A circle is inscribed in a square as shown. The square has perimeter 64 cm. How many square centimeters are in the square but not in the circle (the shaded region of the figure)? Use 3.14 for π and express your answer to the nearest hundredth of a square centimeter.



(3pts) 4. **__55.04_______**

The sides of the square must be 64/4=16 cm. Therefore the area of the square is 16^2 = 256 cm^2. The diameter of the circle must also be 16 cm, so the radius is 8cm and the area is $3.14*8^2$ = 200.96 cm^2. The difference is 55.04 cm^2.

5. What is the greatest prime factor of 2025? (3pts) 5. **____5________**

The usual rules confirm that both 5 and 9 (= 3*3) are factors of 2025. Dividing those into 2025 shows that 2025 = 45^2 = 3*3*3*3*5*5, so 5 is the greatest prime factor of 2025.

6. Four students take a quiz, and each receives a score which is a whole number. The highest score is 10, and the lowest is 2. If the mean (average) score is 8, what is the mode of the scores? (3pts) 6. ____**10**_______

The mean is 8, so the total of the 4 scores must be 8*4 = 32. Since the highest and the lowest scores sum to 12, the middle 2 scores must total 32 – 10 – 2 = 20. Since they must be whole numbers no greater than 10, they must both be 10 as well, and so the three scores of 10 make 10 the mode.

7. The degree measures of the angles of a triangle equal $2x, 3x,$ and $4x$ for some x. What is the measure (in degrees) of the greatest angle? (3pts) 7. ____**80**_______

The sum of the three angles is $2x + 3x + 4x = 9x = 180.$ So $x = 20$, and $4x = 80$.

mm

TOTAL POINTS _____________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #2
Grades 7/8

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Student Name ______________

1. If the diameter of a tire on a vehicle is 16 inches, approximately how far does the vehicle travel in one revolution of the tire?

   A. 20 in. B. 30 in. C. 50 in. D. 60 in.

(2 pts) 1. ________

2. Evaluate the expression: $9 + 4^2 - 4\sqrt{16} + 4 \div 2$

(3 pts) 2. ________

3. Find all solutions for x:

$$x^2 + 70 = 15x^2 + 10$$

(3 pts) 3. ________

4. Given values a and b on a real number line, which expression will always give the distance between a and b ?

   A. $a - b$ B. $b - a$ C. $|a - b|$ D. $|a + b|$

(3 pts) 4. ________

5. The volume of a cubic form is 125 cubic inches. Find the surface area of the cube.

(3 pts) 5. ________

6. Factor completely over reals: $x^4 - 81$

(3 pts) 6. ________

7. Find the one solution (x, y) to the following pair of equations:

$$x - y = 1$$
$$2x + y = 5$$

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #2
Grades 7/8

School ________________ Team Name ________________

Calculators are **NOT** allowed. **Key** Student Name ________________

1. If the diameter of a tire on a vehicle is 16 inches, approximately how far does the vehicle travel in one revolution of the tire?

   A. 20 in. B. 30 in. C. 50 in. D. 60 in.

(2 pts) 1. C

2. Evaluate the expression: $9 + 4^2 - 4\sqrt{16} + 4 \div 2$

(3 pts) 2. 11

3. Find all solutions for x:

$$x^2 + 70 = 15x^2 + 10$$

(3 pts) 3. $x = \pm\frac{\sqrt{210}}{7}$

4. Given values a and b on a real number line, which expression will always give the distance between a and b ?

   A. $a - b$ B. $b - a$ C. $|a - b|$ D. $|a + b|$

(3 pts) 4. C.

5. The volume of a cubic form is 125 cubic inches. Find the surface area of the cube.

(3 pts) 5. 150 in^2

6. Factor completely over reals: $x^4 - 81$

(3 pts) 6. $(x^2+9)(x-3)(x+3)$

7. Find the one solution (x, y) to the following pair of equations:

$$x - y = 1$$
$$2x + y = 5$$

(3 pts) 7. $(2, 1)$

UND MATHEMATICS TRACK MEET | INDIVIDUAL TEST #3
University of North Dakota | Grades 7/8
January 13, 2025

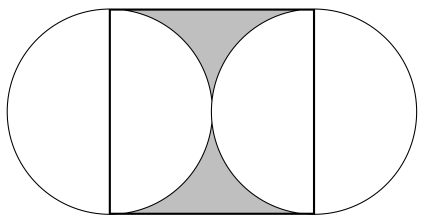
School ______________ Team Name ______________

Calculators are allowed. Student Name ______________

1. Today is a Monday. 150 days from today what day of the week will it be?

(2 pts) 1. ________

2. The square in the figure below has an area of 4 cm^2. What is the area in cm^2 of the shaded region? Round your answer to the nearest hundredths.



(3 pts) 2. ________

3. Two fair six-sided dice are rolled and the numbers on their face are recorded. What is the probability (expressed as a fraction in simplest form) that the sum of the two numbers is a prime?

(3 pts) 3. ________

4. You are given that $\divideontimes$ is an arithmetic operator such that

$$5 \divideontimes 4 = \frac{5}{16}, \quad 2 \divideontimes 10 = \frac{1}{50}, \quad 5 \divideontimes 10 = \frac{1}{20}.$$

What is the value of $2 \divideontimes 4$ expressed as a fraction in simplest form?

(3 pts) 4. ________

5. How much tax (in dollars and cents) is charged on a purchase of \$952.18 if the tax rate is 7.75% for the first \$500 and 3% for the amount over \$500?

(3 pts) 5. ________

6. In a chemistry experiment, the liquid in a container is doubling every 5 minutes. At 60 minutes the container is full. At what time will the container be half full?

(3 pts) 6. ________

7. Ann and Sue buy identical stationary packs with S sheets of paper and E envelopes each. Ann uses her pack to write 1-sheet letters and Sue uses hers to write 3-sheet letters. Ann used all her envelopes and had 50 sheets of paper left, while Sue used all of her sheets of paper and had 50 envelopes left. Find the number of sheets of paper in the stationary pack.

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #3
Grades 7/8

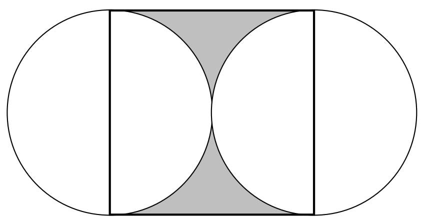
School ______________ Team Name ______________

Calculators are allowed. **Key** Student Name ______________

1. Today is a Monday. 150 days from today what day of the week will it be?

(2 pts) 1. Thursday

2. The square in the figure below has an area of 4 cm^2. What is the area in cm^2 of the shaded region? Round your answer to the nearest hundredths.



(3 pts) 2. $4-\pi = 0.86$

3. Two fair six-sided dice are rolled and the numbers on their face are recorded. What is the probability (expressed as a fraction in simplest form) that the sum of the two numbers is a prime?

(3 pts) 3. $\frac{5}{12}$

4. You are given that $\circledast$ is an arithmetic operator such that

$$5 \circledast 4 = \frac{5}{16}, \quad 2 \circledast 10 = \frac{1}{50}, \quad 5 \circledast 10 = \frac{1}{20}.$$

What is the value of $2 \circledast 4$ expressed as a fraction in simplest form?

(3 pts) 4. $\frac{1}{8}$

5. How much tax (in dollars and cents) is charged on a purchase of \$952.18 if the tax rate is 7.75% for the first \$500 and 3% for the amount over \$500?

(3 pts) 5. 52.32

6. In a chemistry experiment, the liquid in a container is doubling every 5 minutes. At 60 minutes the container is full. At what time will the container be half full?

(3 pts) 6. 55 minutes

7. Ann and Sue buy identical stationary packs with S sheets of paper and E envelopes each. Ann uses her pack to write 1-page letters and Sue uses hers to write 3-page letters. Ann used all her envelopes and had 50 sheets of paper left, while Sue used all of her sheets of paper and had 50 envelopes left. Find the number of sheets of paper in the stationary pack.

(3 pts) 7. 150

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #3
Grades 7/8

School ______________

Team Name ______________

Calculators are allowed.

Solutions

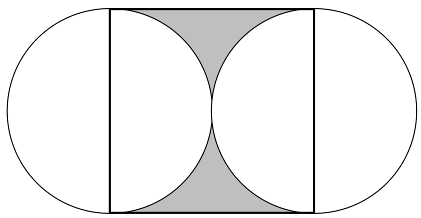
Student Name ______________

1. Today is a Monday. 150 days from today what day of the week will it be?

(2 pts) 1. Thursday

Solution: 147 is the nearest multiple of 7, so 147 days from now will be a Monday. So 150 days from now will be a Thursday.

2. The square in the figure below has an area of 4 cm^2. What is the area in cm^2 of the shaded region? Round your answer to the nearest hundredths.



(3 pts) 2. $4-\pi = 0.86$

Solution: The side of the square, which is also the diameter of the circle is 2 cm. The shaded area is the area of the square minus the area of the circle. That is, $4 - \pi(1)^2 = 4 - \pi = \boxed{0.86}$

3. Two fair six-sided dice are rolled and the numbers on their face are recorded. What is the probability (expressed as a fraction in simplest form) that the sum of the two numbers is a prime?

(3 pts) 3. $\frac{5}{12}$

Solution: The possibilities for the sum of two numbers being prime are
$(1,1),(2,1),(1,2),(3,2),(2,3),(4,1),(1,4),(3,4),(4,3),(5,1),(1,5),(2,5),(5,2),(5,6),(6,5)$.
Therefore the probability is $\frac{15}{36} = \boxed{\frac{5}{12}}$.

4. You are given that $\divideontimes$ is an arithmetic operator such that

$$5 \divideontimes 4 = \frac{5}{16}, \quad 2 \divideontimes 10 = \frac{1}{50}, \quad 5 \divideontimes 10 = \frac{1}{20}.$$

What is the value of $2 \divideontimes 4$ expressed as a fraction in simplest form?

(3 pts) 4. $\frac{1}{8}$

Solution: The operation is $a \divideontimes b = \frac{a}{b^2}$, therefore $2 \divideontimes 4 = \frac{2}{4^2} = \frac{2}{16} = \boxed{\frac{1}{8}}$

5. How much tax (in dollars and cents) is charged on a purchase of \$952.18 if the tax rate is 7.75% for the first \$500 and 3% for the amount over \$500?

(3 pts) 5. 52.32

Solution: The answer is

$$500(0.0775) + (952.18 - 500)(0.03) = 52.3154 \approx \boxed{52.32}$$

6. In a chemistry experiment, the liquid in a container is doubling every 5 minutes. At 60 minutes the container is full. At what time will the container be half full?

(3 pts) 6. 55 minutes

Solution: Since liquid is doubling every 5 minutes and the container is full at 60 minutes, the container will be half full 5 minutes before 60 i.e. at $\boxed{55}$ minutes.

7. Ann and Sue buy identical stationary packs with S sheets of paper and E envelopes each. Ann uses her pack to write 1-sheet letters and Sue uses hers to write 3-sheet letters. Ann used all her envelopes and had 50 sheets of paper left, while Sue used all of her sheets of paper and had 50 envelopes left. Find the number of sheets of paper in the stationary pack.

(3 pts) 7. 150

Solution: Since Ann writes one sheet letters, the number of letters she writes is $S - 50$. The number of envelopes she uses is E and so $E = S - 50$. Sue writes 3-sheet letters and so she writes $S/3$ letters and uses $E - 50$ envelopes. Therefore $S/3 = E - 50$. Solving for S gives $S/3 = S - 100 \Rightarrow 2S = 300 \Rightarrow \boxed{S = 150}$.

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #4
Grades 7/8

School ________________

Team Name ________________

Calculators are **NOT** allowed.

Student Name ________________

1. Evaluate

$$(-100)^0 + 3\left| -4^2 - (-8)\right| + 2\left|\sqrt{5^2 - 3^2} + (-5)^2\right|$$

(2 pts) 1. ________

2. A drama club at Riverdale High School consists only of eleventh grade and twelfth-grade students. If $\frac{2}{5}$ the students in the drama club are from eleventh grade and there are 40 more twelfth-grade students than eleventh-grade students, how many twelfth-grade students are there in the drama club?

(3 pts) 2. ________

3. Two fair six-sided dice are rolled. What is the probability that the sum of the two numbers rolled is at least 8?

(3 pts) 3. ________

4. How many gallons of paint would be needed to paint the walls and the bottom of a rectangular swimming pool that is 40 feet long, 20 feet wide, and 10 feet deep if one gallon of paint covers 250 square feet?

(3 pts) 4. ________

5. A factory makes toy cars using 6 assembly machines. Each machine assembles 15 cars every $\frac{2}{3}$ minutes. How many cars can all 6 machines assemble in two minutes?

(3 pts) 5. ________

6. Find the positive x-intercept, if any, for

$$f(x) = (x+3)^2 - 4$$

(3 pts) 6. ________

7. Simplify the expression

$$\frac{3^4 \times 9^2}{27^3} \times \left(\frac{8^2 \times 16}{4^3}\right)^{\frac{1}{2}}$$

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #4
Grades 7/8

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Key

Student Name ______________

1. Evaluate

$$(-100)^0 + 3\big| - 4^2 - (-8)\big| + 2\big|\sqrt{5^2 - 3^2} + (-5)^2\big|$$

(2 pts) 1. 83

2. A drama club at Riverdale High School consists only of eleventh grade and twelfth-grade students. If $\frac{2}{5}$ the students in the drama club are from eleventh grade and there are 40 more twelfth-grade students than eleventh-grade students, how many twelfth-grade students are there in the drama club?

(3 pts) 2. 120 students

3. Two fair six-sided dice are rolled. What is the probability that the sum of the two numbers rolled is at least 8?

(3 pts) 3. $\frac{15}{36} = \frac{5}{12}$

4. How many gallons of paint would be needed to paint the walls and the bottom of a rectangular swimming pool that is 40 feet long, 20 feet wide, and 10 feet deep if one gallon of paint covers 250 square feet?

(3 pts) 4. 8 gallons

5. A factory makes toy cars using 6 assembly machines. Each machine assembles 15 cars every $\frac{2}{3}$ minutes. How many cars can all 6 machines assemble in two minutes?

(3 pts) 5. 270 cars

6. Find the positive x-intercept, if any, for

$$f(x) = (x + 3)^2 - 4$$

(3 pts) 6. None

7. Simplify the expression

$$\frac{3^4 \times 9^2}{27^3} \times \left(\frac{8^2 \times 16}{4^3}\right)^{\frac{1}{2}}$$

(3 pts) 7. $\frac{4}{3}$

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

TEAM TEST #1
Grades 7/8

School ______________ Team Name ______________

Calculators are allowed.

1. What is the radius of a circle having the same area as a rectangle with sides of length 11 and 13?

(20 pts) 1. ________

2. The area of a triangle is 150. If the base of the triangle is three times the height, what is the height?

(20 pts) 2. ________

3. Find positive numbers r and s with $rs = 81$ and $r + s = 20$.

(20 pts) 3. ________

4. In a class your grade is determined by the average of five exam scores. If the average of your first two exams is 77%, what must the average of your last three exams be to get an 85% in the class?

(20 pts) 4. ________

5. A bacterial culture doubles every 12 minutes. There are $30,720$ bacteria at noon. How many bacteria were there at 10 am earlier that same day?

(20 pts) 5. ________

6. What is the greatest common divisor of 360 and 525?

(20 pts) 6. ________

7. Find the positive solution to $2^{x^2-3x} = 16 \cdot 8^{x+5}$.

(20 pts) 7. ________

8. Suppose the price of an item is increased by 20% and then later the new price is decreased by 10%. Compared to the original price, what percentage increase does the final price represent?

(20 pts) 8. ________

9. Solve the system of equations

$$\begin{cases} x + 2y = 2025 \\ 3x + 5y = 1776 \end{cases}$$

(20 pts) 9. ________

10. Suppose a and b are positive real numbers such that $a - b = 5$ and $ab = 2$. What is $a + b$?

(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET — TEAM TEST #1

University of North Dakota — Grades 7/8

January 13, 2025

School ______________ **Key** Team Name ______________

Calculators are allowed.

1. What is the radius of a circle having the same area as a rectangle with sides of length 11 and 13?

(20 pts) 1. 6.75

2. The area of a triangle is 150. If the base of the triangle is three times the height, what is the height?

(20 pts) 2. 10

3. Find positive numbers r and s with $rs = 81$ and $r + s = 20$.

(20 pts) 3. $\{5.64, 14.36\}$

4. In a class your grade is determined by the average of five exam scores. If the average of your first two exams is 77%, what must the average of your last three exams be to get an 85% in the class?

(20 pts) 4. $90\frac{1}{3}\%$

5. A bacterial culture doubles every 12 minutes. There are 30,720 bacteria at noon. How many bacteria were there at 10 am earlier that same day?

(20 pts) 5. 30

6. What is the greatest common divisor of 360 and 525?

(20 pts) 6. 15

7. Find the positive solution to $2^{x^2-3x} = 16 \cdot 8^{x+5}$.

(20 pts) 7. $3+2\sqrt{7} \approx 8.29$

8. Suppose the price of an item is increased by 20% and then later the new price is decreased by 10%. Compared to the original price, what percentage increase does the final price represent?

(20 pts) 8. 8%

9. Solve the system of equations

$$\begin{cases} x + 2y &= 2025 \\ 3x + 5y &= 1776 \end{cases}$$

(20 pts) 9. $x = -6573, y = 4299$

10. Suppose a and b are positive real numbers such that $a - b = 5$ and $ab = 2$. What is $a + b$?

(20 pts) 10. $\sqrt{33} \approx 5.74$

January 2025 Math Track Meet 7-8 Team Test 1 (with calculator) -AJB

1. What is the radius of a circle having the same area as a rectangle with sides of length 11 and 13?

   [Sketch of solution: We must have $\pi r^2 = 11 \cdot 13$ and so $r = \sqrt{143/\pi} \approx 6.75$.]

2. The area of a triangle is 150. If the base of the triangle is three times the height, what is the height?

   [Sketch: We have $A = \frac{1}{2}bh$ and $b = 3h$. So solve $150 = \frac{1}{2} \cdot 3h \cdot h$ to find $h = 10$.]

3. Find positive numbers r and s with $rs = 81$ and $r + s = 20$.

   [Sketch: Since $(x - r)(x - s) = x^2 - 20x + 81$, the quadratic formula gives $\{r, s\} \approx \{5.64, 14.36\}$.]

4. In a class your grade is determined by the average of five exam scores. If the average of your first two exams is 77%, what must the average of your last three exams be to get an 85% in the class?

   [Sketch: If a, b are the first two exam scores and c, d, e are the last three, we have $(a + b)/2 = 77$ and $(a + b + c + d + e)/5 = 85$. Solve to get $c + d + e = 271$, and so $(c + d + e)/3 = 90\frac{1}{3}$.]

5. A bacterial culture doubles every 12 minutes. There are 30,720 bacteria at noon. How many bacteria were there at 10 am earlier that same day?

   [Sketch: The population doubles 10 times between 10 am and noon so the number of bacteria at 10 am is $30720/2^{10} = 30$.]

6. What is the greatest common divisor of 360 and 525?

   [Sketch: We have $360 = 2^3 \cdot 3^2 \cdot 5$ and $525 = 3 \cdot 5^2 \cdot 7$ so the GCD is $3 \cdot 5 = 15$.]

7. Find the positive solution to $2^{x^2-3x} = 16 \cdot 8^{x+5}$.

   [Sketch: We have $2^{x^2-3x} = 2^{4+3x+15}$, and so $x^2 - 6x - 19 = 0$. The solutions are $3 \pm 2\sqrt{7}$. The positive solution is $3 + 2\sqrt{7} \approx 8.29$.]

8. Suppose the price of an item is increased by 20% and then later the new price is decreased by 10%. Compared to the original price, what percentage increase does the final price represent?

   [Sketch: If P is the original price, the final price is $(.9)(1.20)P = (1.08)P$. So the price has increased by 8%.]

9. Solve the system of equations

$$\begin{cases} x + 2y &= 2025 \\ 3x + 5y &= 1776 \end{cases}$$

[Sketch: Use Gaussian elimination or substitution to find $x = -6573$, $y = 4299$.]

10. Suppose a and b are positive real numbers such that $a - b = 5$ and $ab = 2$. What is $a + b$?

[Sketch: Since $(a+b)^2 = (a-b)^2 + 4ab = 33$ we have $a + b = \sqrt{33} \approx 5.74$.]

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

TEAM TEST #2
Grades 7/8

School ______________

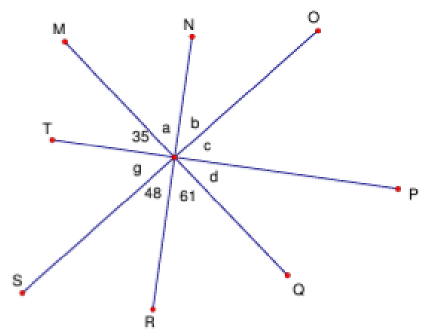
Team Name ______________

Calculators are **NOT** allowed.

1. What is 2025% of 2025?

(20 pts) 1. ________

2. In the diagram to the right, the four segments: $\overline{MQ}, \overline{NR}, \overline{OS}, \overline{PT}$ all intersect at the same point creating 8 angles labeled with either their angle measures or variable names. What is the value of $a + b + c + d + g$?



(20 pts) 2. ________

3. A game consists of black and white pieces. The number of black pieces is 4 more than 3 times the number of white pieces. Seven white and 15 black pieces are removed each round. After several rounds, there are 3 white and 55 black pieces left. How many pieces were there in the beginning?

(20 pts) 3. ________

4. A tournament had six players. Each player played every other player only once, with no ties. If Helen won 4 games, Ines won 3 games, Janet won 2 games, Kendra won 2 games, and Lara won 2 games, how many games did Monica win?

(20 pts) 4. ________

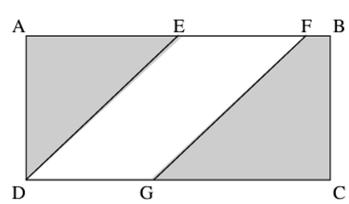
5. I have a container with $1\frac{5}{6}$ cups of flour. The container is $\frac{1}{3}$ full. How much flour does the container have when it is full?

(20 pts) 5. ________

6. The average of two whole numbers is 18 and their product is 308. What is the positive difference between the two numbers?

(20 pts) 6. ________

7. In the rectangle $ABCD$, the measure of $\overline{AB}$ is 24 inches, the measure of $\overline{AD}$ is 11 inches, and the measure of $\overline{EF}$ is 10 inches. If $\overline{ED}$ is parallel to $\overline{FG}$, how many square inches are in the combined area of the shaded regions.



(20 pts) 7. ________

8. All of the shirts in Joel's closet are either gray t-shirts or plaid flannels. Two-fifths of his shirts are flannels. If he were to buy another flannel, then $\frac{3}{7}$ of his shirts would be flannels. How many t-shirts are there in Joel's closet?

(20 pts) 8. ________

9. The length of each side of a parallelogram is multiplied by 6 to create a similar parallelogram whose area is 612 ft^2. How many square feet are in the area of the smaller parallelogram?

(20 pts) 9. ________

10. Five marbles are in a bowl. Two are green and 3 are black. What is the probability of picking a green marble out of the bowl on the first draw (and not returning it to the bowl) and a black marble on the second draw?

(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

TEAM TEST #2
Grades 7/8

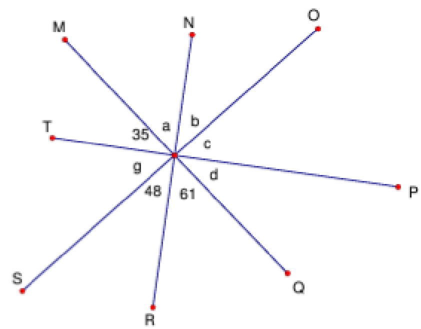
School ______________ **Key** Team Name ______________

Calculators are **NOT** allowed.

1. What is 2025% of 2025?

(20 pts) 1. 41,006.25

2. In the diagram to the right, the four segments: $\overline{MQ}, \overline{NR}, \overline{OS}, \overline{PT}$ all intersect at the same point creating 8 angles labeled with either their angle measures or variable names. What is the value of $a + b + c + d + g$?



(20 pts) 2. 216

3. A game consists of black and white pieces. The number of black pieces is 4 more than 3 times the number of white pieces. Seven white and 15 black pieces are removed each round. After several rounds, there are 3 white and 55 black pieces left. How many pieces were there in the beginning?

(20 pts) 3. 212

4. A tournament had six players. Each player played every other player only once, with no ties. If Helen won 4 games, Ines won 3 games, Janet won 2 games, Kendra won 2 games, and Lara won 2 games, how many games did Monica win?

(20 pts) 4. 2

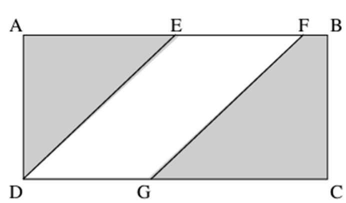
5. I have a container with $1\frac{5}{6}$ cups of flour. The container is $\frac{1}{3}$ full. How much flour does the container have when it is full?

(20 pts) 5. $5\frac{1}{2}$ cups

6. The average of two whole numbers is 18 and their product is 308. What is the positive difference between the two numbers?

(20 pts) 6. 8

7. In the rectangle $ABCD$, the measure of $\overline{AB}$ is 24 inches, the measure of $\overline{AD}$ is 11 inches, and the measure of $\overline{EF}$ is 10 inches. If $\overline{ED}$ is parallel to $\overline{FG}$, how many square inches are in the combined area of the shaded regions.



(20 pts) 7. 154 in^2

8. All of the shirts in Joel's closet are either gray t-shirts or plaid flannels. Two-fifths of his shirts are flannels. If he were to buy another flannel, then $\frac{3}{7}$ of his shirts would be flannels. How many t-shirts are there in Joel's closet?

(20 pts) 8. 12

9. The length of each side of a parallelogram is multiplied by 6 to create a similar parallelogram whose area is 612 ft^2. How many square feet are in the area of the smaller parallelogram?

(20 pts) 9. 17

10. Five marbles are in a bowl. Two are green and 3 are black. What is the probability of picking a green marble out of the bowl on the first draw (and not returning it to the bowl) and a black marble on the second draw?

(20 pts) 10. $\frac{3}{10}$

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #1
Grades 9/10

School ________

Team Name ________

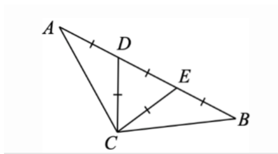
Calculators are allowed.

Student Name ________

1. The average of a list of 10 numbers is 17. When one number is removed from the list, the new average is 16. What number was removed?

(2 pts) 1. ________

2. In $\triangle ABC$, points D and E lie on AB, as shown. If $AD = DE = EB = CD = CE$, What is the measure of $\angle ABC$ (in degrees)?



(3 pts) 2. ________

3. A numerical value is assigned to each letter of the alphabet. The value of a word is determined by adding up the numerical values of each of its letters. The value of SET is 2, the value of HAT is 7, the value of TASTE is 3, and the value of MAT is 4. What is the value of the word MATH?

(3 pts) 3. ________

4. There are 20 students in a class. In total, 10 of them have black hair, 5 of them wear glasses, and 3 of them both have black hair and wear glasses. How many of the students have black hair but do not wear glasses?

(3 pts) 4. ________

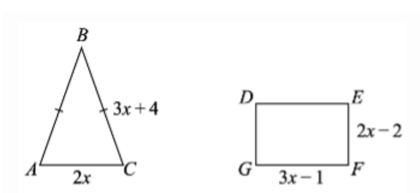
5. A hiker is exploring a trail. The trail has three sections: the first 25% of the trail is along a river, the next 5/8 of the trail is through a forest, and the remaining 3 km of the trail is up a hill. How long is the trail?

(3 pts) 5. ________

6. The first 2 hours of Melanie's trip was spent travelling at 100 km/h. The remaining 200 km of Melanie's trips was spent travelling at 80 km/h. What is Melanie's average speed during this trip (rounded to 2 decimal places)?

(3 pts) 6. ________

7. In the diagram, $\triangle ABC$ has $AB = BC = 3x + 4$ and $AC = 2x$ and rectangle $DEFG$ has $EF = 2x - 2$ and $FG = 3x - 1$. The perimeter of $\triangle ABC$ is equal to the perimeter of rectangle $DEFG$. What is the area of $\triangle ABC$?



(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #1
Grades 9/10

School ________________

Team Name ________________

Calculators are allowed.

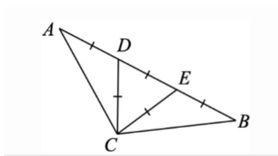
Key

Student Name ________________

1. The average of a list of 10 numbers is 17. When one number is removed from the list, the new average is 16. What number was removed?

(2 pts) 1. 26

2. In $\triangle ABC$, points D and E lie on AB, as shown. If $AD = DE = EB = CD = CE$, What is the measure of $\angle ABC$ (in degrees)?



(3 pts) 2. 30

3. A numerical value is assigned to each letter of the alphabet. The value of a word is determined by adding up the numerical values of each of its letters. The value of SET is 2, the value of HAT is 7, the value of TASTE is 3, and the value of MAT is 4. What is the value of the word MATH?

(3 pts) 3. 10

4. There are 20 students in a class. In total, 10 of them have black hair, 5 of them wear glasses, and 3 of them both have black hair and wear glasses. How many of the students have black hair but do not wear glasses?

(3 pts) 4. 7

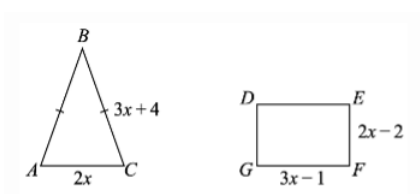
5. A hiker is exploring a trail. The trail has three sections: the first 25% of the trail is along a river, the next 5/8 of the trail is through a forest, and the remaining 3 km of the trail is up a hill. How long is the trail?

(3 pts) 5. 24 km

6. The first 2 hours of Melanie's trip was spent travelling at 100 km/h. The remaining 200 km of Melanie's trips was spent travelling at 80 km/h. What is Melanie's average speed during this trip (rounded to 2 decimal places)?

(3 pts) 6. 88.89 km/h

7. In the diagram, $\triangle ABC$ has $AB = BC = 3x + 4$ and $AC = 2x$ and rectangle $DEFG$ has $EF = 2x - 2$ and $FG = 3x - 1$. The perimeter of $\triangle ABC$ is equal to the perimeter of rectangle $DEFG$. What is the area of $\triangle ABC$?



(3 pts) 7. 168

FORKS AREA MATHEMATICS TRACK MEET — INDIVIDUAL TEST #1
University of North Dakota — Grades 9/10
January 2024

SOLUTIONS

1.

When 10 numbers have an average of 17, their sum is $10 \times 17 = 170$.
When 9 numbers have an average of 16, their sum is $9 \times 16 = 144$.
Therefore, the number that was removed was $170 - 144 = 26$.

2.

Since $CD = DE = EC$, then $\triangle CDE$ is equilateral, which means that $\angle DEC = 60^\circ$.
Since $\angle DEB$ is a straight angle, then $\angle CEB = 180^\circ - \angle DEC = 180^\circ - 60^\circ = 120^\circ$.
Since $CE = EB$, then $\triangle CEB$ is isosceles with $\angle ECB = \angle EBC$.
Since $\angle ECB + \angle CEB + \angle EBC = 180^\circ$, then $2 \times \angle EBC + 120^\circ = 180^\circ$, which means that $2 \times \angle EBC = 60^\circ$ or $\angle EBC = 30^\circ$.
Therefore, $\angle ABC = \angle EBC = 30^\circ$.

3.

From the given information, we know that

$$\begin{aligned} \mathrm{S+E+T} &= 2 \\ \mathrm{H+A+T} &= 7 \\ \mathrm{T+A+S+T+E} &= 3 \\ \mathrm{M+A+T} &= 4 \end{aligned}$$

Since $\mathrm{T+A+S+T+E} = 3$ and $\mathrm{S+E+T} = 2$, then $\mathrm{T+A} = 3 - 2 = 1$.
Since $\mathrm{H+A+T} = 7$ and $\mathrm{T+A} = 1$, then $\mathrm{H} = 7 - 1 = 6$.
Since $\mathrm{M+A+T} = 4$ and $\mathrm{H} = 7$, then $\mathrm{M} + (\mathrm{A+T}) + \mathrm{H} = 4 + 6 = 10$.
Therefore, the value of the word MATH is 10.
We note that it is also possible to find specific values for S, E, T, A that give the correct values to the words. One such set of values is $\mathrm{A} = 1$, $\mathrm{T} = 0$, $\mathrm{S} = 4$, and $\mathrm{E} = -2$. These values are not unique, even though the values assigned to M and H (namely, 3 and 6) are unique.

4.

Since 10 students have black hair and 3 students have black hair and wear glasses, then a total of $10 - 3 = 7$ students have black hair but do not wear glasses.

5.

Since 25% is equivalent to $\frac{1}{4}$, then the fraction of the trail covered by the section along the river and the section through the forest is $\frac{1}{4} + \frac{5}{8} = \frac{2}{8} + \frac{5}{8} = \frac{7}{8}$.
This means that the final section up a hill represents $1 - \frac{7}{8} = \frac{1}{8}$ of the trail.
Since $\frac{1}{8}$ of the trail is $3\ \mathrm{km}$ long, then the entire trail is $8 \times 3\ \mathrm{km} = 24\ \mathrm{km}$ long.

6.

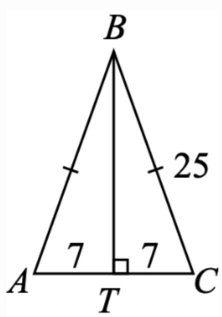
In 2 hours travelling at 100 km/h, Melanie travels $2 \text{ h} \times 100 \text{ km/h} = 200 \text{ km}$.
When Melanie travels 200 km at 80 km/h, it takes $\dfrac{200 \text{ km}}{80 \text{ km/h}} = 2.5 \text{ h}$.
Melanie travels a total of $200 \text{ km} + 200 \text{ km} = 400 \text{ km}$.
Melanie travels for a total of $2 \text{ h} + 2.5 \text{ h} = 4.5 \text{ h}$.
Therefore, Melanie's average speed is $\dfrac{400 \text{ km}}{4.5 \text{ h}} \approx 88.89 \text{ km/h}$.

7.

The perimeter of $\triangle ABC$ is equal to $(3x+4)+(3x+4)+2x = 8x+8$.
The perimeter of rectangle $DEFG$ is equal to

$$2 \times (2x-2) + 2 \times (3x-1) = 4x - 4 + 6x - 2 = 10x - 6$$

Since these perimeters are equal, we have $10x - 6 = 8x + 8$ which gives $2x = 14$ and so $x = 7$.
Thus, $\triangle ABC$ has $AC = 2 \times 7 = 14$ and $AB = BC = 3 \times 7 + 425$.
We drop a perpendicular from B to T on AC.



Since $\triangle ABC$ is isosceles, then T is the midpoint of AC, which gives $AT = TC = 7$.
By the Pythagorean Theorem, $BT = \sqrt{BC^2 - TC^2} = \sqrt{25^2 - 7^2} = \sqrt{625 - 49} = \sqrt{576} = 24$.
Therefore, the area of $\triangle ABC$ is equal to $\frac{1}{2} \cdot AC \cdot BT = \frac{1}{2} \times 14 \times 24 = 168$.

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #2
Grades 9/10

School ________________ Team Name ________________

Calculators are **NOT** allowed. Student Name ________________

1. What is $x + \frac{1}{x}$ if $x^2 = 2 - \frac{1}{x^2}$?

(2 pts) 1. ________

2. If $-2 \leq x \leq 1, -1 \leq y \leq 3$, then $a \leq x^2y \leq 12$. What is a?

(3 pts) 2. ________

3. $x = 1$ is a root of $x^3 - x^2 + ax + 4 = 0$. What are the other roots?

(3 pts) 3. ________

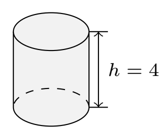
4. If (a, b) is the minimum point on the graph of $y = x^2 - 6x + 10$, what is the distance of (a, b) from the y-axis?

(3 pts) 4. ________

5. If $\log_{10}(x^2 - 3) > 0$, then $x < a$ or $x > b$. What is a?

(3 pts) 5. ________

6. The height of a circular cylinder with top and bottom lids is 4 cm. If the surface area and the volume are the same, what is the radius of the cylinder?



(3 pts) 6. ________

7. In a survey of 100 people, 50 liked baseball, 70 liked basketball and 20 did not like either sport. How many people liked both baseball and basketball?

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #2
Grades 9/10

School ______________ Team Name ______________

Calculators are **NOT** allowed. **Key** Student Name ______________

1. What is $x + \dfrac{1}{x}$ if $x^2 = 2 - \dfrac{1}{x^2}$?

(2 pts) 1. ± 2

2. If $-2 \le x \le 1, -1 \le y \le 3$, then $a \le x^2 y \le 12$. What is a?

(3 pts) 2. -4

3. $x = 1$ is a root of $x^3 - x^2 + ax + 4 = 0$. What are the other roots?

(3 pts) 3. ± 2

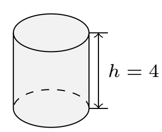
4. If (a, b) is the minimum point on the graph of $y = x^2 - 6x + 10$, what is the distance of (a, b) from the y-axis?

(3 pts) 4. 3

5. If $\log_{10}(x^2 - 3) > 0$, then $x < a$ or $x > b$. What is a?

(3 pts) 5. -2

6. The height of a circular cylinder with top and bottom lids is 4 cm. If the surface area and the volume are the same, what is the radius of the cylinder?



(3 pts) 6. 4

7. In a survey of 100 people, 50 liked baseball, 70 liked basketball and 20 did not like either sport. How many people liked both baseball and basketball?

(3 pts) 7. 40

9th Individual #2 w/o calculators. (DH)

1. $x^2 + 2 + 1/x^2 = 4 \implies (x + 1/x)^2 = 4 \implies x + 1/x = \pm 2.$ — 1. $\underline{\pm 2}$

2. $0 \le x^2 \le 4$ and $-1 \le y \le 3 \implies -4 \le x^2 y \le 12.$ — 2. $\underline{-4}$

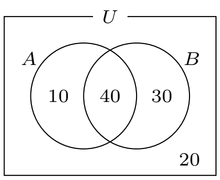
3. $a = -4 \implies x^3 - x^2 - 4x + 4 = (x-1)(x^2-4) \implies x = 1, \pm 2.$ — 3. $\underline{\pm 2}$

4. $y = (x-3)^2 + 1 \implies (a, b) = (3, 1).$ — 4. $\underline{3}$

5. $x^2 - 3 > 1 \implies x < -2$ or $x > 2.$ — 5. $\underline{-2}$

6. $4\pi r^2 = 2\pi r^2 + 8\pi r \implies r^2 - 4r = r(r-4) = 0.$ — 6. $\underline{4}$

7. $n(U) = 100,\ n(A) = 50,\ n(B) = 70,\ n(U - A \cup B) = n(U) - n(A \cup B) = 20$
$\implies n(A \cup B) = 80 = n(A) + n(B) - n(A \cap B) = 120 - n(A \cap B).$ — 7. $\underline{40}$



UND MATHEMATICS TRACK MEET | INDIVIDUAL TEST #3
University of North Dakota | Grades 9/10
January 13, 2025

School ______________ Team Name ______________

Calculators are allowed. Student Name ______________

1. Find the two numbers that add to 20 and multiply to 91.

(2 pts) 1. ________

2. How many (x, y) points have integer coordinates and are *exactly* 5 units from the origin?

(3 pts) 2. ________

3. How many positive integers evenly divide 2025, including 1 and 2025?

(3 pts) 3. ________

4. A standard deck of 52 cards has 13 values in each of 4 different suits. How many ways are there to get only one suit in a hand of 5 cards?

(3 pts) 4. ________

5. Find the smallest natural number n so that the sum of the integers from 1 to n (inclusive) is at least 2025.

(3 pts) 5. ________

6. A quadratic function of the form $f(x) = ax^2 + c$ has $f(3) = 5$ and $f(5) = 3$. What is $f(-3)$?

(3 pts) 6. ________

7. Write down 2025 copies of the letter A. Beginning from the left, replace every pair of A's with a single B, so that there is one A left over. Repeat the process, replacing pairs of B's with C's, pairs of C's with D's, and so on. When you cannot replace any more, what is left?

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #3
Grades 9/10

ANSWER KEY
School ______________

ANSWER KEY
Team Name ______________

Calculators are allowed.

Student Name ______________

1. Find the two numbers that add to 20 and multiply to 91.

(2 pts) 1. 7 and 13

2. How many (x, y) points have integer coordinates and are *exactly* 5 units from the origin?

(3 pts) 2. 12

3. How many positive integers evenly divide 2025, including 1 and 2025?

(3 pts) 3. 15

4. A standard deck of 52 cards has 13 values in each of 4 different suits. How many ways are there to get only one suit in a hand of 5 cards?

(3 pts) 4. $4\binom{13}{5} = 5148$

5. Find the smallest natural number n so that the sum of the integers from 1 to n (inclusive) is at least 2025.

(3 pts) 5. 64

6. A quadratic function of the form $f(x) = ax^2 + c$ has $f(3) = 5$ and $f(5) = 3$. What is $f(-3)$?

(3 pts) 6. 5

7. Write down 2025 copies of the letter A. Beginning from the left, replace every pair of A's with a single B, so that there is one A left over. Repeat the process, replacing pairs of B's with C's, pairs of C's with D's, and so on. When you cannot replace any more, what is left?

(3 pts) 7. KJIHGFDA

TOTAL POINTS ______________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #3
Grades 9/10

SOLUTION ANSWER KEY
School ______________

SOLUTION ANSWER KEY
Team Name ______________

Calculators are allowed.

Student Name ______________

1. Find the two numbers that add to 20 and multiply to 91.
Solution: $x + y = 20$ and $xy = 91$ means that $x(20 - x) = 91$, or $0 = x^2 - 20x + 91 = (x - 7)(x - 13)$.

(2 pts) 1. 7 and 13

2. How many (x, y) points have integer coordinates and are *exactly* 5 units from the origin?
Solution: The four points $(\pm3, \pm4)$, the four points $(\pm4, \pm3)$, the two points $(0, \pm5)$, and the two points $(\pm5, 0)$.

(3 pts) 2. 12

3. How many positive integers evenly divide 2025, including 1 and 2025?
Solution: Since $2025 = 3^4 \cdot 5^2$, there are $(4 + 1)(2 + 1)$ factors.

(3 pts) 3. 15

4. A standard deck of 52 cards has 13 values in each of 4 different suits. How many ways are there to get only one suit in a hand of 5 cards?
Solution: There are four options for the suit and $\binom{13}{5}$ options for the values.

(3 pts) 4. $4\binom{13}{5} = 5148$

5. Find the smallest natural number n so that the sum of the integers from 1 to n (inclusive) is at least 2025.
Solution: We need $n(n+1)/2 \geq 2025$, so $n(n+1) = 4050$. Since this is about 2^{12}, we can start by checking 2^6, seeing that $64 \cdot 65/2 = 2080 \geq 2025$, and $63 \cdot 64/2 = 2016 < 2025$.

(3 pts) 5. 64

6. A quadratic function of the form $f(x) = ax^2 + c$ has $f(3) = 5$ and $f(5) = 3$. What is $f(-3)$?
Solution: The function is even. Alternatively, $9a+c = 5$ and $25a+c = 3$ means $16a = -2$ so that $a = -1/8$ and $c = 49/8$.

(3 pts) 6. 5

7. Write down 2025 copies of the letter A. Beginning from the left, replace every pair of A's with a single B, so that there is one A left over. Repeat the process, replacing pairs of B's with C's, pairs of C's with D's, and so on. When you cannot replace any more, what is left?
Solution: This is equivalent to finding the binary representation of 2025. We have

$$A^{2025} \to B^{1012}A \to C^{506}A \to D^{253}A \to E^{126}DA \to F^{63}DA \to G^{31}FDA$$
$$\to H^{15}GFDA \to I^{7}HGFDA \to J^{3}IHGFDA \to KJIHGFDA$$

(3 pts) 7. KJIHGFDA

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #4
Grades 9/10

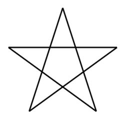
School ______________

Team Name ______________

Calculators are **NOT** allowed.

Student Name ______________

1. For the figure below, how many rotations and reflections carry it onto itself? Do not count the reflection or rotation that leaves all points in the same place.



(2 pts) 1. ________

2. You are driving to your cabin that is x miles away. On the first weekend, you could travel 75 mph. The next weekend there was a snowstorm, and you could only travel 50 mph to your cabin. The second weekend took you 42 min longer to travel than the first. How far away is your cabin?

(3 pts) 2. ________

3. A moving truck rental company charges \$50 to rent a truck for 10 miles and \$80 for 25 miles. Assume the truck rental charges are linear with miles. How much will the company charge for you rent a truck for 85 miles?

(3 pts) 3. ________

4. Determine the length, l of a rectangle, with width w, if its perimeter is 44, its area is 105, and $l > w$.

(3 pts) 4. ________

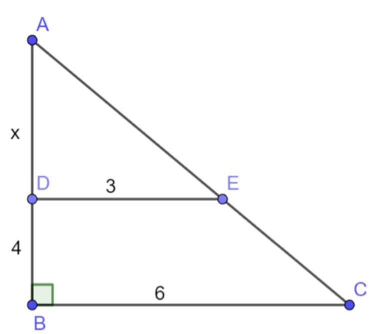
5. A number is divided by 3 and then 15 is added to the result to give 46. What is the original number?

(3 pts) 5. ________

6. Each of the numbers 1, 2, 3, and 4 is substituted, in some order, for p, q, r, and s. What is the highest possible value of $p^q + r^s$?

(3 pts) 6. ________

7. Find the side length x.



(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #4
Grades 9/10

School ______________

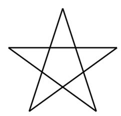
Team Name ______________

Calculators are **NOT** allowed.

Key

Student Name ______________

1. For the figure below, how many rotations and reflections carry it onto itself? Do not count the reflection or rotation that leaves all points in the same place.



(3 pts) 1. 9

2. You are driving to your cabin that is x miles away. On the first weekend, you could travel 75 mph. The next weekend there was a snowstorm, and you could only travel 50 mph to your cabin. The second weekend took you 42 min longer to travel than the first. How far away is your cabin?

(3 pts) 2. 105 miles

3. A moving truck rental company charges \$50 to rent a truck for 10 miles and \$80 for 25 miles. Assume the truck rental charges are linear with miles. How much will the company charge for you rent a truck for 85 miles?

(3 pts) 3. \$200

4. Determine the length, l of a rectangle, with width w, if its perimeter is 44, its area is 105, and $l > w$.

(3 pts) 4. 15

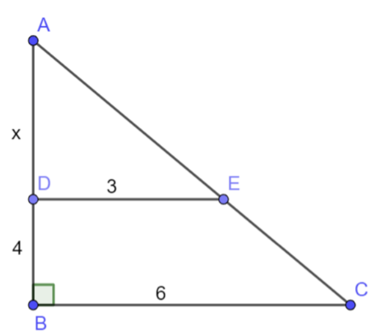
5. A number is divided by 3 and then 15 is added to the result to give 46. What is the original number?

(3 pts) 5. 93

6. Each of the numbers 1, 2, 3, and 4 is substituted, in some order, for p, q, r, and s. What is the highest possible value of $p^q + r^s$?

(3 pts) 6. 83

7. Find the side length x.



(3 pts) 7. 4

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

TEAM TEST #1
Grades 9/10

School ______________

Team Name ______________

Calculators are allowed.

1. Determine exactly the solution to

$$\frac{3x+2}{x+5} - \frac{8x+6}{3x+15} = 1$$

(20 pts) 1. ________

2. At a school dance class attended by 200 juniors and seniors, the seniors are asked to teach the juniors how to dance. Anna danced with 7 juniors, Joey danced with 8 juniors, Hailey danced with 9 juniors, and so on through the last senior, who danced with all of the juniors. How many juniors attended the dance?

(20 pts) 2. ________

3. a, b, and c are positive integers such that the sum of a and b is 3 more than the value of c, the value of a is double that of b, and the value of b is five less than c. Determine exactly the value of b.

(20 pts) 3. ________

4. The base of a triangular flower bed is 4 meters longer than its height. If the area of the flower bed is 30 m^2, find the height of the triangle.

(20 pts) 4. ________

5. Find the sum of the two solutions to this absolute value equation: $|2x+1| = 9$

(20 pts) 5. ________

6. When the height of a triangle is quadrupled, its area increased by 2025 cm^2. What is the area of the original triangle?

(20 pts) 6. ________

7. Five years ago, Maria was three times as old as Alex. Five years from now, Maria will be twice as old as Alex. What is Alex's current age?

(20 pts) 7. ________

8. In 2023, the number of total enrolled students at the University of North Dakota was $14,172$. This student population represented approximiately 24% of the population of Grand Forks. About how many people lived in Grand Forks in 2023? Round your answer to the nearest integer.

(20 pts) 8. ________

9. (x, y) is the intersection of $\frac{3x}{y} - 4 = 11$ and $x - y = 44$. Determine the value of $x + y$.

(20 pts) 9. ________

10. Summing the powers of 3 produces an interesting pattern that allows you to find the sum without adding. Discover the pattern and use it to find the sum of: $1 + 3 + 9 + 27 + \cdots + 531441$.

$$1 = 1$$
$$1 + 3 = 4$$
$$1 + 3 + 9 = 13$$
$$1 + 3 + 9 + 27 = 40$$
$$1 + 3 + 9 + 27 + 81 = \underline{\quad}$$
$$1 + 3 + 9 + 27 + 81 + 243 = \underline{\quad}$$

(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

TEAM TEST #1
Grades 9/10

School ______________ **Key** Team Name ______________

Calculators are allowed.

1. Determine exactly the solution to

$$\frac{3x+2}{x+5} - \frac{8x+6}{3x+15} = 1$$

(20 pts) 1. $x = -\frac{15}{2}$

2. At a school dance class attended by 200 juniors and seniors, the seniors are asked to teach the juniors how to dance. Anna danced with 7 juniors, Joey danced with 8 juniors, Hailey danced with 9 juniors, and so on through the last senior, who danced with all of the juniors. How many juniors attended the dance?

(20 pts) 2. 103

3. a, b, and c are positive integers such that the sum of a and b is 3 more than the value of c, the value of a is double that of b, and the value of b is five less than c. Determine exactly the value of b.

(20 pts) 3. 4

4. The base of a triangular flower bed is 4 meters longer than its height. If the area of the flower bed is 30 m^2, find the height of the triangle.

(20 pts) 4. 6

5. Find the sum of the two solutions to this absolute value equation: $|2x+1| = 9$

(20 pts) 5. -1

6. When the height of a triangle is quadrupled, its area increased by 2025 cm^2. What is the area of the original triangle?

(20 pts) 6. 675

7. Five years ago, Maria was three times as old as Alex. Five years from now, Maria will be twice as old as Alex. What is Alex's current age?

(20 pts) 7. 15

8. In 2023, the number of total enrolled students at the University of North Dakota was 14,172. This student population represented approximiately 24% of the population of Grand Forks. About how many people lived in Grand Forks in 2023? Round your answer to the nearest integer.

(20 pts) 8. 59,050

9. (x, y) is the intersection of $\frac{3x}{y} - 4 = 11$ and $x - y = 44$. Determine the value of $x + y$.

(20 pts) 9. 66

10. Summing the powers of 3 produces an interesting pattern that allows you to find the sum without adding. Discover the pattern and use it to find the sum of: $1 + 3 + 9 + 27 + \cdots + 531441$.

$$1 = 1$$
$$1 + 3 = 4$$
$$1 + 3 + 9 = 13$$
$$1 + 3 + 9 + 27 = 40$$
$$1 + 3 + 9 + 27 + 81 = \_\_\_$$
$$1 + 3 + 9 + 27 + 81 + 243 = \_\_\_$$

(20 pts) 10. 797161

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

TEAM TEST #1
Grades 9/10

School ________________ **Solutions** Team Name ________________

Calculators are allowed.

1. Determine exactly the solution to

$$\frac{3x+2}{x+5} - \frac{8x+6}{3x+15} = 1$$

> $$1 = \frac{3x+2}{x+5} - \frac{8x+6}{3x+15} = \frac{9x+6}{3x+15} - \frac{8x+6}{3x+15} = \frac{x}{3x+15}$$
> $$\Rightarrow x = 3x+15 \Rightarrow -2x = 15 \Rightarrow x = -\frac{15}{2}$$

2. At a school dance class attended by 200 juniors and seniors, the seniors are asked to teach the juniors how to dance. Anna danced with 7 juniors, Joey danced with 8 juniors, Hailey danced with 9 juniors, and so on through the last senior, who danced with all of the juniors. How many juniors attended the dance?

> The first senior danced with $6+1$ juniors, the second senior danced with $6+2$ juniors, etc., so we can say that the n^{th} senior danced with $6+n$ juniors. Since there were n seniors and $6+n$ juniors at the dance,
> $n + \left(6+n\right) = 200 \Rightarrow 2n+6 = 200 \Rightarrow n = 97$, so there were 97 seniors and 103 juniors.

3. a, b, and c are positive integers such that the sum of a and b is 3 more than the value of c, the value of a is double that of b, and the value of b is five less than c. Determine exactly the value of b.

> If the sum of a and b is 3 more than c, then $a+b = c+3$. Since a is double b and b is five less than c, then $a = 2b$ and $b = c-5$ respectively. Substituting the second and third equations into the first gives us:
>
> $$2(c-5) + (c-5) = c+3 \Rightarrow 3(c-5) = c+3 \Rightarrow 2c = 18 \Rightarrow c = 9$$
>
> Therefore, $b = 9-5 = 4$

4. The base of a triangular flower bed is 4 meters longer than its height. If the area of the flower bed is 30 m^2, find the height of the triangle.

Let the height of the triangle be h. Then, the base is $h+4$. Substitute into the formula for the area of a triangle:

$$\begin{aligned}30 &= \frac{1}{2} \cdot (h+4) \cdot h \\ 30 &= \frac{1}{2}(h^2+4h) \\ 60 &= h^2+4h \\ 0 &= h^2+4h-60 \\ 0 &= (h+10)(h-6)\end{aligned}$$

Solving for h, we get $h=-10$ and $h=6$, but height here cannot be negative, so our height must be 6 meters.

5. Find the sum of the two solutions to this absolute value equation: $|2x+1|=9$

We get two equations to solve, either $2x+1=9$ or $2x+1=-9$. In the first case, $x=4$. In the second case, $x=-5$. Their sum is -1.

6. When the height of a triangle is quadrupled, its area increased by 2025 cm^2. What is the area of the original triangle?

$$\frac{1}{2}b(4h) - \frac{1}{2}bh = 2025 \Rightarrow \frac{3}{2}bh = 2025 \Rightarrow \frac{1}{2}bh = \frac{2025}{3} = 675 \text{ cm}^2$$

7. Five years ago, Maria was three times as old as Alex. Five years from now, Maria will be twice as old as Alex. What is Alex's current age?

Let Alex's current age be x and Maria's current age be y.

1. Five years ago:
   Maria's age was $y-5$, and Alex's age was $x-5$. From the problem:
   $$y - 5 = 3(x-5).$$
2. Five years from now:
   Maria's age will be $y+5$, and Alex's age will be $x+5$. From the problem:
   $$y + 5 = 2(x+5).$$
3. Solve the system of equations:
   From the first equation:
   $$y - 5 = 3x - 15 \quad \Rightarrow \quad y = 3x - 10.$$
   Substitute $y = 3x - 10$ into the second equation:
   $$(3x - 10) + 5 = 2(x+5).$$
   Simplify:
   $$3x - 5 = 2x + 10.$$
   Solve for x:
   $$x = 15.$$

Hence, Alex is currently 15 years old.

8. In 2023, the number of total enrolled students at the University of North Dakota was 14,172. This student population represented approximiately 24% of the population of Grand Forks. About how many people lived in Grand Forks in 2023? Round your answer to the nearest integer.

Let the total population of Grand Forks in 2023 be x. We know that 24% of this total is the number of enrolled students at the University of North Dakota, which is 14,172. This relationship can be written as:
$$0.24x = 14,172.$$
To solve for x, divide both sides of the equation by 0.24:
$$x = \frac{14,172}{0.24}.$$
Simplify the division:
$$x = 59,050.$$
Hence, the population of Grand Forks in 2023 was approximately 59,050 people.

9. (x, y) is the intersection of $\frac{3x}{y} - 4 = 11$ and $x - y = 44$. Determine the value of $x + y$.

$3x = 15y \Rightarrow x = 5y$. Therefore, $5y - y = 44 \Rightarrow y = 11$. So, $x = 55$ and $x + y = 66$

10.

Summing the powers of 3 produces an interesting pattern that allows you to find the sum without adding. Discover the pattern and use it to find the sum of: $1 + 3 + 9 + 27 + \cdots + 531441$.

$$1 = 1$$
$$1 + 3 = 4$$
$$1 + 3 + 9 = 13$$
$$1 + 3 + 9 + 27 = 40$$
$$1 + 3 + 9 + 27 + 81 = \_\_\_$$
$$1 + 3 + 9 + 27 + 81 + 243 = \_\_\_$$

Take the last power of three, muliply by 3, subract 1, and divide by 2.
Step 1:

$$531441 \cdot 3 = 1594323$$

Step 2:

$$1594323 - 1 = 1594322$$

Step 3:

$$\frac{1594322}{2} = 797161$$

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

TEAM TEST # 2
Grades 9/10

School ______________ Team Name ______________

Calculators are **NOT** allowed.

1. A movie theater in a small town usually open its doors 3 days in a row and then closes the next day for maintenance. Another movie theater is open 4 days in a row and then closes the next day for the same reason. Suppose both movie theaters are closed today and that today is Wednesday. What day is it the next time they are both closed again at the same time?

(20 pts) 1. ________

2. Let ℓ_1 be the line segment whose endpoints lie at the top left and bottom right corner of a square and let ℓ_2 be the line segment whose endpoints are the bottom left corner of the square and the center of the top side of the square. What fraction of the area of the square is not occupied by the triangle formed from the point at the intersection of ℓ_1 and ℓ_2 and the bottom two corners of the square?

(20 pts) 2. ________

3. The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5050 is collected. How many more children than adults attended?

(20 pts) 3. ________

4. The sum of the squares of two consecutive positive odd integers is 74. What is the value of the larger integer?

(20 pts) 4. ________

5. How many ways are there to pick two nonconsecutive numbers from the first 50 positive integers?

(20 pts) 5. ________

6. Suppose the rational function $f(x)$ is given by $f(x) = \dfrac{5x+7}{2x+4}$. Find $f^{-1}(x)$.

(20 pts) 6. ________

7. A parallelogram with sides of length x and $\dfrac{3x+2}{2}$ has perimeter 32. What is the length of the longer side?

(20 pts) 7. ________

8. If the cost of a bat and a baseball combined is \$1.10 and the ball costs \$1.00 less than the bat, how much is the ball?

(20 pts) 8. ________

9. The square root of the value obtained by adding twelve to some positive number is the same as the number itself. What is it?

(20 pts) 9. ________

10. What is the third smallest two digit positive integer that is equal to seven times the sum of its digits?

(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

TEAM TEST # 2
Grades 9/10

School ______________ **Key** Team Name ______________

Calculators are **NOT** allowed.

1. A movie theater in a small town usually open its doors 3 days in a row and then closes the next day for maintenance. Another movie theater is open 4 days in a row and then closes the next day for the same reason. Suppose both movie theaters are closed today and that today is Wednesday. What day is it the next time they are both closed again at the same time?

(20 pts) 1. Tuesday

2. Let ℓ_1 be the line segment whose endpoints lie at the top left and bottom right corner of a square and let ℓ_2 be the line segment whose endpoints are the bottom left corner of the square and the center of the top side of the square. What fraction of the area of the square is not occupied by the triangle formed from the point at the intersection of ℓ_1 and ℓ_2 and the bottom two corners of the square?

(20 pts) 2. $\frac{2}{3}$

3. The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5050 is collected. How many more children than adults attended?

(20 pts) 3. 800

4. The sum of the squares of two consecutive positive odd integers is 74. What is the value of the larger integer?

(20 pts) 4. 7

5. How many ways are there to pick two nonconsecutive numbers from the first 50 positive integers?

(20 pts) 5. 1176

6. Suppose the rational function $f(x)$ is given by $f(x) = \dfrac{5x+7}{2x+4}$. Find $f^{-1}(x)$.

(20 pts) 6. $f^{-1}(x) = \dfrac{7-4x}{2x-5}$

7. A parallelogram with sides of length x and $\dfrac{3x+2}{2}$ has perimeter 32. What is the length of the longer side?

(20 pts) 7. 10

8. If the cost of a bat and a baseball combined is \$1.10 and the ball costs \$1.00 less than the bat, how much is the ball?

(20 pts) 8. \$0.05

9. The square root of the value obtained by adding twelve to some positive number is the same as the number itself. What is it?

(20 pts) 9. 4

10. What is the third smallest two digit positive integer that is equal to seven times the sum of its digits?

(20 pts) 10. 63

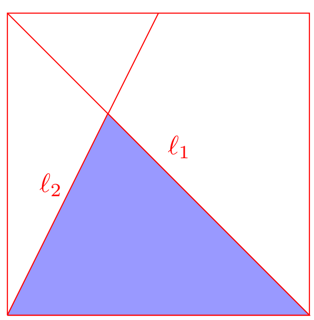
UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

TEAM TEST # 2
Grades 9/10

Answer Key

1. A movie theater in a small town usually opens its doors 3 days in a row and then closes the next day for maintenance. Another movie theater is open 4 days in a row and then closes the next day for the same reason. Suppose both movie theaters are closed today and that today is Wednesday. What day is it the next time they are both closed again at the same time?
The first theater has an open-closed cycle that repeats every 4 days, while the second has an open-closed cycle that repeats every 5 days. The least common multiple of 4 and 5 is 20, so we need only note that 20 days after Wednesday is Tuesday. That is, the next time both theaters will be closed at once will occur on a Tuesday.

2. Let ℓ_1 be the line segment whose endpoints lie at the top left and bottom right corner of a square and let ℓ_2 be the line segment whose endpoints are the bottom left corner of the square and the center of the top side of the square. What fraction of the area of the square is not occupied by the triangle formed from the point at the intersection of ℓ_1 and ℓ_2 and the bottom two corners of the square?
Consider a square of side length 1 for simplicity, so that the area of the triangle is the same as the proportion of the square's area that it occupies. The triangle formed is shaded in blue below:



Notice that the height of the triangle and the triangle formed above it sum to 1, and that these two triangles are similar. Let h be the height of the larger triangle, so that $1-h$ is the height of the smaller. As the ratio of the height to the base of similar triangles is the same, we then have

$$\frac{h}{1} = \frac{1-h}{1/2}$$
$$h = 2 - 2h$$
$$h = \frac{2}{3}.$$

So, the area of the triangle is $\frac{1}{2}(1)(h) = \frac{1}{3}$, which is also the proportion of the area of any square a triangle formed in such a fashion will occupy. Hence, the proportion of the square not occupied by the triangle is $\frac{2}{3}$.

3. The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5050 is collected. How many

more children than adults attended?
Let x denote the number of adults and let y denote the number of children. Then, we have the system

$$\begin{cases} x + y &= 2200 \\ 4x + 1.5y &= 5050 \end{cases}$$

So, we solve the system for x and y using whatever method seems easiest. Here, we opt for substitution and solve the first equation for x:

$$\begin{aligned} x + y &= 2200 \\ x &= 2200 - y \end{aligned}$$

Now, we substitute our formula for x in to the second equation

$$\begin{aligned} 4x + 1.5y &= 5050 \\ 4(2200 - y) + 1.5y &= 5050 \\ 8800 - 2.5y &= 5050 \\ y &= \frac{5050 - 8800}{-2.5} = 1500. \end{aligned}$$

Using our equation from the first step, this gives $x = 2200 - 1500 = 700$. So, we have our answer: 700 adults and 1500 children attended the fair, and $1500 - 700 = 800$ more children attended.

4. The sum of the squares of two consecutive positive odd integers is 74. What is the value of the larger integer?
Every odd integer is of the form $2k + 1$ for some integer $k \geq 0$. So, the sum of the squares of any two consecutive odd integers is of the form $(2k+1)^2 + (2k+3)^2 = 8k^2 + 16k + 10$. If the value of this sum is to be 74, then we need only solve $8k^2 + 16k + 10 = 74$, which amounts to finding the roots of the quadratic $8k^2 + 16k - 64 = 0$, or (factoring out an 8) $k^2 + 2k - 8 = 0$. Via the quadratic formula, this produces $k = -4$ or $k = 2$. Since we are looking for a positive odd integer, we may conclude that $k = 2$. Thus, the smaller integer is $2(2) + 1 = 5$ and the larger is $2(2) + 3 = 7$.

5. How many ways are there to pick two nonconsecutive numbers from the first 50 positive integers?
There are $\binom{50}{2} = \frac{50!}{2!(50-2)!} = \frac{(50)(49)(48!)}{(2)(48!)} = (25)(49) = 1225$ ways to choose two numbers from the first 50 positive integers. As a set of two consecutive integers $\{x, x+1\}$ is determined uniquely by the smaller of the two values (i.e. by x), notice that there are $50 - 1 = 49$ possible choices for this smallest value (as $50 + 1 = 51$ is not in the first 50 positive integers). So, there are 49 ways to choose two consecutive numbers from the first 50 positive integers, and any of the other 1225 possible ways to choose two numbers from the first 50 positive integers other than these 49 ways results in a pair of nonconsecutive values. Thus, we reach our answer: There are $1225 - 49 = 1176$ ways to pick two nonconsecutive numbers from the first 50 positive integers.

6. Suppose the rational function $f(x)$ is given by $f(x) = \frac{5x+7}{2x+4}$. Find $f^{-1}(x)$.
Writing $y = f(x)$, we have $y = \frac{5x+7}{2x+4}$. To find $f^{-1}(x)$, we interchange x and y

and solve for y:

$$\begin{aligned} x &= \frac{5y+7}{2y+4} \\ x(2y+4) &= 5y+7 \\ 2xy+4x &= 5y+7 \\ 2xy-5y &= 7-4x \\ (2x-5)y &= 7-4x \\ y &= \frac{7-4x}{2x-5}. \end{aligned}$$

So, replacing y with $f^{-1}(x)$, we have found our inverse function:

$$f^{-1}(x) = \frac{7-4x}{2x-5}.$$

7. A parallelogram with sides of length x and $\frac{3x+2}{2}$ has perimeter 32. What is the length of the longer side?
   The perimeter of a parallelogram with sides of length x and $\frac{3x+2}{2}$ is $2(x) + 2\left(\frac{3x+2}{2}\right) = 5x+2$. Therefore, we may conclude that $5x+2 = 32$ and solve for x to produce $x = 6$. Correspondingly, the sides of the parallelogram are of length $x = 6$ and $\frac{3x+2}{2} = \frac{3(6)+2}{2} = 10$. Thus, the longer side is of length 10.

8. If the cost of a bat and a baseball combined is \$1.10 and the ball costs \$1.00 less than the bat, how much is the ball?
   Let x denote the cost of the bat and y denote the cost of the ball. We know that $x + y = \$1.10$ and $x = y + \$1.00$. Combining the second equation with the first, we produce

$$\begin{aligned} x+y &= \$1.10 \\ (y+\$1.00)+y &= \$1.10 \\ 2y+\$1.00 &= \$1.10 \\ 2y &= \$0.10 \\ y &= \$0.05. \end{aligned}$$

   Therefore, the ball costs \$0.05.

9. The square root of the value obtained by adding twelve to some positive number is the same as the number itself. What is it?
   Let x denote our sought value. Then, $\sqrt{x+12} = x$, and we have

$$\begin{aligned} \sqrt{x+12} &= x \\ x+12 &= x^2 \\ x^2-x-12 &= 0 \\ x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-12)}}{2(1)} \\ x &= \frac{1 \pm \sqrt{1+48}}{2} \\ x &= \frac{1 \pm 7}{2} \\ x &= 4 \text{ or } -3 \end{aligned}$$

   As the number is specified to be positive, we may conclude that it is 4.

10. What is the third smallest two digit integer that is equal to seven times the sum of its digits?
Write a two-digit integer as $10a + b$, where $a, b \in \{0, \ldots, 9\}$. If $10a + b = 7(a + b)$, then $10a + b = 7a + 7b$, and so $3a = 6b$, or, more simply, $a = 2b$. That is, the second digit must be twice the first. The smallest such positive number is 21. The next is $42 = 7(2 + 4)$ and the third is $63 = 7(6 + 3)$.

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #1
Grades 11/12

School ______________

Team Name ______________

Calculators are allowed.

Student Name ______________

1. Three cards are chosen at random from a standard 52-card deck. What is the probability that they are not all the same color? Express your answer as a simple fraction in lowest terms.

(2 pts) 1. ________

2. Find the exact value of
$$\frac{2^{-2024}+2^{-2022}}{2^{-2020}+2^{-2018}}$$
Express your answer as a simple fraction in lowest terms.

(3 pts) 2. ________

3. What is the exact value of $x+y$ for real numbers x and y satisfying the following equation?
$$x^2+y^2=10x-6y-34$$

(3 pts) 3. ________

4. The real numbers a, b, and c form an arithmetic sequence with $a \geq b \geq c \geq 0$. The quadratic ax^2+bx+c has exactly one root. What is this root?

(3 pts) 4. ________

5. In a triangle ΔABC, the angles satisfy the ratio $\angle A : \angle B : \angle C = 1 : 2 : 3$. If the longest side of ΔABC is 10 units, what is the perimeter of the triangle ΔABC?

(3 pts) 5. ________

6. Line L_1 has the equation $3x-2y=1$ and passes through the point $A(-1,-2)$. Line L_2 is given by $y=1$ and intersects L_1 at the point B. Line L_3 , which has a positive slope, passes through point A and intersects L_2 at the point C. The area of ΔABC is 3. What is the slope of L_3?

(3 pts) 6. ________

7. Let $f_1(x)=10x-1$ and $f_n(x)=f_1\left(f_{n-1}(x)\right)$ for integers $n \geq 2$. What is the sum of the digits of $f_{2025}(1)$?

(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #1
Grades 11/12

School ______________ Team Name ______________

Calculators are allowed. Student Name ______________

1. Three cards are chosen at random from a standard 52-card deck. What is the probability that they are not all the same color? Express your answer as a simple fraction in lowest terms.

(2 pts) 1. $\frac{13}{17}$

2. Find the exact value of
$$\frac{2^{-2024}+2^{-2022}}{2^{-2020}+2^{-2018}}$$
Express your answer as a simple fraction in lowest terms.

(3 pts) 2. $\frac{1}{16}$

3. What is the exact value of $x+y$ for real numbers x and y satisfying the following equation?
$$x^2+y^2=10x-6y-34$$

(3 pts) 3. 2

4. The real numbers a, b, and c form an arithmetic sequence with $a \geq b \geq c \geq 0$. The quadratic ax^2+bx+c has exactly one root. What is this root?

(3 pts)4. $\frac{-1}{2+\sqrt{3}}$ or $\sqrt{3}-2$

5. In a triangle ΔABC, the angles satisfy the ratio $\angle A : \angle B : \angle C = 1:2:3$. If the longest side of ΔABC is 10 units, what is the perimeter of the triangle ΔABC?

(3 pts) 5. $15+5\sqrt{3}$

6. Line L_1 has the equation $3x-2y=1$ and passes through the point $A(-1,-2)$. Line L_2 is given by $y=1$ and intersects L_1 at the point B. Line L_3 , which has a positive slope, passes through point A and intersects L_2 at the point C. The area of ΔABC is 3. What is the slope of L_3?

(3 pts) 6. $\frac{3}{4}$

7. Let $f_1(x)=10x-1$ and $f_n(x)=f_1\left(f_{n-1}(x)\right)$ for integers $n \geq 2$. What is the sum of the digits of $f_{2025}(1)$?

(3 pts) 7. 16,201

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET | INDIVIDUAL TEST #1
University of North Dakota | Grades 11/12
January 13, 2025

School ______________ Team Name ______________

Calculators are allowed. Student Name ______________

1. Three cards are chosen at random from a standard 52-card deck. What is the probability that they are not all the same color? Express your answer as a simple fraction in lowest terms.

(2 pts) 1. $\frac{13}{17}$

Solution Find the probability they are all same color, then subtract that from 1. There are 26 cards of each color, so 3 of them can be selected in $\binom{26}{3}$ ways, and there are 2 colors. So the answer is $1 - \frac{2\binom{26}{3}}{\binom{52}{3}} = \boxed{\frac{13}{17}}$.

2. Find the exact value of

$$\frac{2^{-2024} + 2^{-2022}}{2^{-2020} + 2^{-2018}}$$

Express your answer as a simple fraction in lowest terms.

(3 pts) 2. $\frac{1}{16}$

Solution $\frac{2^{-2024}(1+2^2)}{2^{-2020}(1+2^2)} = 2^{-4} = \boxed{\frac{1}{16}}$

3. What is the exact value of $x + y$ for real numbers x and y satisfying the following equation?

$$x^2 + y^2 = 10x - 6y - 34$$

(3 pts) 3. 2

Solution The equation can be expressed $(x-5)^2 + (y+3)^2 = 0$. Then $x = 5$ and $y = -3$. So $x + y = \boxed{2}$.

4. The real numbers a, b, and c form an arithmetic sequence with $a \geq b \geq c \geq 0$. The quadratic $ax^2 + bx + c$ has exactly one root. What is this root?

(3 pts)4. $\frac{-1}{2+\sqrt{3}}$ or $\sqrt{3} - 2$

Solution Let $a = b+k$ and $c = b-k$ for some $k > 0$. Since the quadratic has exactly one root, $b^2 - 4ac = 0 \implies b^2 - 4(b+k)(b-k) = -3b^2 + 4k^2 = 0 \implies k = \frac{\sqrt{3}}{2}b$. Then the root is $-\frac{b}{2a} = -\frac{b}{2\left(b + \frac{\sqrt{3}}{2}b\right)} = \boxed{\frac{-1}{2+\sqrt{3}}} = \boxed{\sqrt{3} - 2}$

5. In a triangle ΔABC, the angles satisfy the ratio $\angle A : \angle B : \angle C = 1 : 2 : 3$. If the longest side of ΔABC is 10 units, what is the perimeter of the triangle ΔABC?

(3 pts) 5. $15 + 5\sqrt{3}$

Solution Let $\angle A = x$, $\angle B = 2x$, and $\angle C = 3x$. Then $x + 2x + 3x = 180° \implies x = 30°$. So the anges are $\angle A = 30°$, $\angle B = 60°$, $\angle C = 90°$. So ΔABC is a right trangle. Then, the side lengths are 5 units, $5\sqrt{3}$ units, and 10 units. Thus the perimeter is $\boxed{15 + 5\sqrt{3}}$ units.

6. Line L_1 has the equation $3x - 2y = 1$ and passes through the point $A(-1, -2)$. Line L_2 is given by $y = 1$ and intersects L_1 at the point B. Line L_3 , which has a positive slope, passes through point A and intersects L_2 at the point C. The area of ΔABC is 3. What is the slope of L_3?

(3 pts) 6. $\frac{3}{4}$

Solution The points A, B, and C form a triangle. The distance from the point A to L_2 is 3, which is the height of the trangle, and the length of the line segment between B and C should be 2. Since L_3 has a positive slope, the point C is at the point $(3, 1)$. Then the slope of L_3 is $\boxed{\dfrac{3}{4}}$.

7. Let $f_1(x) = 10x - 1$ and $f_n(x) = f_1(f_{n-1}(x))$ for integers $n \geq 2$. What is the sum of the digits of $f_{2025}(1)$?

(3 pts) 7. 16,201

Solution $f_2(x) = 10(10x-1)-1 = 10^2x - 10 - 1$, $f_3(x) = 10^3x - 10^2 - 10 - 1, \cdots,$ $f_{2025}(x) = 10^{2025}x - 10^{2024} - \cdots - 10 - 1$. Then $f_{2025}(1) = 8888\cdots89$ with 2024 eights and one nine. Thus, the sum of the digits is $8 * 2024 + 9 = \boxed{16,201}$.

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #2
Grades 11/12

School ______________

Team Name ______________

Calculators are **NOT** allowed.

Student Name ______________

1. Two rectangles have the same perimeter. In the first rectangle the ratio of the long edge to the short edge is 2 : 1 and in the second rectangle these edges are in the ratio of 3 : 2. Then the ratio of the area of the first rectangle to the area of the second rectangle is

   (a) 2 : 3 (b) 25 : 6 (c) 25 : 27 (d) 27 : 2 (e) not determined

   (2 pts) 1. ________

2. How many pairs of integers (x, y) satisfy the equation $xy - 2x - 3y + 1 = 0$?

   (a) 1 (b) 2 (c) 3 (d) 4 (e) infinitely many

   (3 pts) 2. ________

3. Given square $ABCD$ with side length 4, points E, F, G, and H are taken on sides AB, BC, CD, and DA, respectively, such that $AE = BF = CG = DH$. Determine the length of AE that minimizes the perimeter of quadrilateral $EFGH$.

   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

   (3 pts) 3. ________

4. Let f be the function such that $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ satisfying $f(-1) = 0$ and $f(x) - f(x - 1) = 2x(x + 1)(2x + 1)$ for any real number x. What is the value of $2025a + 13b + c$?

   (a) 2025 (b) 2042 (c) 2082 (d) 2100 (e) 2185

   (3 pts) 4. ________

5. Chris's garden is twice as large as Sarah's garden and three times as large as James's. James waters plants half as quickly as Sarah and one-third as quickly as Chris. If they all start watering their gardens at the same time, who will finish watering first?

   (a) Chris (b) Sarah (c) James and Chris tie for first
   (d) Sarah and James tie for first (e) None of these

   (3 pts) 5. ________

6. Peter has 250 students enrolled in his school club. He decides to form groups so that each group has a different number of students. The maximum number of groups he can form is:

   (a) 20 (b) 21 (c) 22 (d) 23 (e) none of these

   (3 pts) 6. ________

7. For how many integers $m \geq 0$ is $m^2 + 7$ a perfect square?

   (a) 1 (b) 2 (c) 3 (d) 4 (e) no integer is satisfied

   (3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #2
Grades 11/12

School ______________ Team Name ______________

Calculators are **NOT** allowed. **Key** Student Name ______________

1. Two rectangles have the same perimeter. In the first rectangle the ratio of the long edge to the short edge is 2 : 1 and in the second rectangle these edges are in the ratio of 3 : 2. Then the ratio of the area of the first rectangle to the area of the second rectangle is

   (a) 2 : 3 (b) 25 : 6 (c) 25 : 27 (d) 27 : 2 (e) not determined

(2 pts) 1. (c)

2. How many pairs of integers (x, y) satisfy the equation $xy - 2x - 3y + 1 = 0$?

   (a) 1 (b) 2 (c) 3 (d) 4 (e) infinitely many

(3 pts) 2. (d)

3. Given square $ABCD$ with side length 4, points E, F, G, and H are taken on sides AB, BC, CD, and DA, respectively, such that $AE = BF = CG = DH$. Determine the length of AE that minimizes the perimeter of quadrilateral $EFGH$.

   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

(3 pts) 3. (b)

4. Let f be the function such that $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ satisfying $f(-1) = 0$ and $f(x) - f(x-1) = 2x(x+1)(2x+1)$ for any real number x. What is the value of $2025a + 13b + c$?

   (a) 2025 (b) 2042 (c) 2082 (d) 2100 (e) 2185

(3 pts) 4. (c)

5. Chris's garden is twice as large as Sarah's garden and three times as large as James's. James waters plants half as quickly as Sarah and one-third as quickly as Chris. If they all start watering their gardens at the same time, who will finish watering first?

   (a) Chris (b) Sarah (c) James and Chris tie for first
   (d) Sarah and James tie for first (e) None of these

(3 pts) 5. (b)

6. Peter has 250 students enrolled in his school club. He decides to form groups so that each group has a different number of students. The maximum number of groups he can form is:

   (a) 20 (b) 21 (c) 22 (d) 23 (e) none of these

(3 pts) 6. (b)

7. For how many integers $m \geq 0$ is $m^2 + 7$ a perfect square?

   (a) 1 (b) 2 (c) 3 (d) 4 (e) no integer is satisfied

(3 pts) 7. (a)

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET — INDIVIDUAL TEST #2

University of North Dakota — Grades 11/12

January 13, 2025

School ______________ Team Name ______________

Calculators are **NOT** allowed. **Solutions** Student Name ______________

1. Two rectangles have the same perimeter. In the first rectangle the ratio of the long edge to the short edge is 2 : 1 and in the second rectangle these edges are in the ratio of 3 : 2. Then the ratio of the area of the first rectangle to the area of the second rectangle is

   (a) 2 : 3 (b) 25 : 6 (c) 25 : 27 (d) 27 : 2 (e) not determined

   (2 pts) 1. (c)

   Solution. Let x and y represent the long and short edges of the first rectangle, respectively, while z and t represent the long and short edges of the second rectangle. We have $x = 2y$, $z = \frac{3}{2}t$, and $x + y = z + t$, which implies that $3y = \frac{5}{2}t$, or $\frac{y}{t} = \frac{5}{6}$. The ratio of the area of the first rectangle to the area of the second rectangle is

$$\frac{xy}{zt} = \frac{2y^2}{\frac{3}{2}t^2} = \frac{4}{3}.\left(\frac{5}{6}\right)^2 = \frac{25}{27}.$$

2. How many pairs of integers (x, y) satisfy the equation $xy - 2x - 3y + 1 = 0$?

   (a) 1 (b) 2 (c) 3 (d) 4 (e) infinitely many

   (3 pts) 2. (d)

   Solution. We can rewrite the above equation as

$$y(x-3) = 2x - 1 \iff y = \frac{2x-1}{x-3} = 2 + \frac{5}{x-3}.$$

   Since y is an integer, 5 is divisible by $x - 3$, which implies that

$$x - 3 \in \{1, -1, 5, -5\} \iff x \in \{-2, 2, 4, 8\}.$$

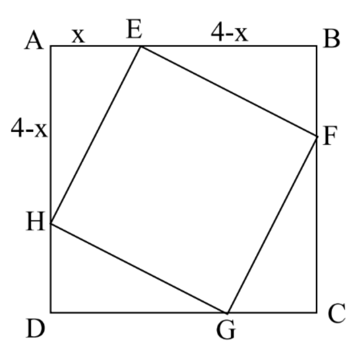
   Therefore, there are 4 pairs of integers (x, y) satisfying the equation in question.

3. Given square $ABCD$ with side length 4, points E, F, G, and H are taken on sides AB, BC, CD, and DA, respectively, such that $AE = BF = CG = DH$. Determine the length of AE that minimizes the perimeter of quadrilateral $EFGH$.

   (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

   (3 pts) 3. (b)

   Solution.



It is not difficult to verify that $EFGH$ is a square. Let $AE = x$, then we must have $AH = 4 - x$, which implies by the Pythagorean theorem that

$$EH^2 = x^2 + (4-x)^2 = 2x^2 - 8x + 16 = 2(x-2)^2 + 8 \geq 8.$$

Therefore, the perimeter of $EFGH$ attains minimum value if and only if $x = 2$.

4. Let f be the function such that $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ satisfying $f(-1) = 0$ and $f(x) - f(x-1) = 2x(x+1)(2x+1)$ for any real number x. What is the value of $2025a + 13b + c$? (a) 2025 (b) 2042 (c) 2082 (d) 2100 (e) 2185

(3 pts) 4. (c)

Solution. Because $f(-1) = 0$ and $f(x) - f(x-1) = 2x(x+1)(2x+1)$, we deduce that

$$f(0) = f(-1) = 0, \qquad f(-1) = f(-2) = 0.$$

Therefore, we can write

$$f(x) = x(x+1)(x+2)(mx+n),$$

for some real numbers m and n. Again, using the fact that $f(x) - f(x-1) = 2x(x+1)(2x+1)$, we have

$$f(1) - f(0) = 12, \qquad f(2) - f(1) = 60,$$

or

$$6m + 6n = 12, \quad 42m + 18n = 60,$$

which implies that $m = n = 1$. Therefore,

$$f(x) = x(x+1)(x+2)(x+1) = x^4 + 4x^3 + 5x^2 + 2x.$$

This means that $a = 1, b = 4, c = 5, d = 2, e = 0$. So, (c) is the correct answer.

5. Chris's garden is twice as large as Sarah's garden and three times as large as James's. James waters plants half as quickly as Sarah and one-third as quickly as Chris. If they all start watering their gardens at the same time, who will finish watering first?

(a) Chris (b) Sarah (c) James and Chris tie for first
(d) Sarah and James tie for first (e) None of these

(3 pts) 5. (b)

Solution. Let the size of James's garden be x. Then:

- Chris's garden size is $3x$,
- Sarah's garden size is $\frac{3x}{2}$.

The watering rates are:

- James: r,
- Sarah: $2r$,
- Chris: $3r$.

The time to water each garden is:

$$\text{James: } \frac{x}{r}, \quad \text{Sarah: } \frac{\frac{3x}{2}}{2r} = \frac{3x}{4r}, \quad \text{Chris: } \frac{3x}{3r} = \frac{x}{r}.$$

Clearly, $\frac{3x}{4r} < \frac{x}{r}$, so Sarah will finish watering first.

6. Peter has 250 students enrolled in his school club. He decides to form groups so that each group has a different number of students. The maximum number of groups he can form is:

(a) 20 (b) 21 (c) 22 (d) 23 (e) none of these

(3 pts) 6. (b)

Solution. To find the maximum number of groups Peter can form such that each group has a different number of students and the total number of students is 250, we solve the inequality:

$$\frac{n(n+1)}{2} \le 250.$$

Rewriting:

$$n(n+1) \le 500.$$

Solving the quadratic equation $n^2 + n - 500 = 0$ using the quadratic formula:

$$n = \frac{-1 \pm \sqrt{1 + 4 \cdot 500}}{2} = \frac{-1 \pm \sqrt{2001}}{2}.$$

Note that $\sqrt{2001} < \sqrt{2025} = 45$, we get:

$$n < \frac{-1 + 45}{2} = 22.$$

Thus, the largest integer n is 21.

7. For how many integers $m \ge 0$ is $m^2 + 7$ a perfect square?

(a) 1 (b) 2 (c) 3 (d) 4 (e) no integer is satisfied

(3 pts) 7. (a)

Solution. $m^2 + 7$ is a perfect square if it can be written as

$$m^2 + 7 = n^2 \iff n^2 - m^2 = 7,$$

for some integer n. This is equivalent to

$$(n - m)(n + m) = 7.$$

We have the following possibilities:

$$n - m = 1, n + m = 7 \iff n = 4, m = 3,$$

$$n - m = 7, n + m = 1 \iff n = 4, m = -3,$$

$$n - m = -1, n + m = -7 \iff n = -4, m = -3,$$

$$n - m = -7, n + m = -1 \iff n = -4, m = 3.$$

In conclusion, there is only 1 number $m \ge 0$ satisfying the requirement of the question. So, (a) is the correct answer.

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #3
Grades 11/12

School ______________

Team Name ______________

Calculators are allowed.

Student Name ______________

1. Alex needs to create a four-digit code for his locker. The digits must be selected from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and they must be distinct. To make it easier to remember, Alex decides to arrange the digits in strictly decreasing order. For instance, the code 9530 follows this rule. How many different four-digit codes can Alex choose?

(2 pts) 1. ________

2. Elsa shopped at a store where each item costs either \$1.00 or 49 cents. She spent a total of \$39.84 How many items did she purchase?

(3 pts) 2. ________

3. What is the coefficient of the x^3y^7 term in the expansion of $(2x + y)^{10}$?

(3 pts) 3. ________

4. If you draw four lines in the plane, they divide the plane into some number of regions. What is the maximum number of regions you can get?

(3 pts) 4. ________

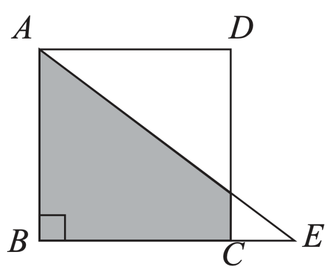
5. What is the smallest positive integer x such that x^2 has 2024 as a factor?

(3 pts) 5. ________

6. The arithmetic mean A, of any two positive numbers x and y is defined to be $A = \frac{1}{2}(x + y)$ and their geometric mean, G is defined to be $G = \sqrt{xy}$. For two particular values x and y, with $x > y$, the ratio $A : G = 5 : 4$. For these values of x and y, what is the ratio $x : y$?

(3 pts) 6. ________

7. The diagram shows a square $ABCD$ and a right-angled triangle ABE. The length of BC is 4 and the length of BE is 5. What is the area of the shaded region?



(3 pts) 7. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #3
Grades 11/12

School ______________

Team Name ______________

Calculators are allowed.

Key

Student Name ______________

1. Alex needs to create a four-digit code for his locker. The digits must be selected from the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and they must be distinct. To make it easier to remember, Alex decides to arrange the digits in strictly decreasing order. For instance, the code 9530 follows this rule. How many different four-digit codes can Alex choose?

(3 pts) 1. 210

2. Elsa shopped at a store where each item costs either \$1.00 or 49 cents. She spent a total of \$39.84 How many items did she purchase?

(3 pts) 2. 32+16 = 48

3. What is the coefficient of the x^3y^7 term in the expansion of $(2x + y)^{10}$?

(3 pts) 3. 960

4. If you draw four line in the plane, they divide the plane into some number of regions. What is the maximum number of regions you can get?

(3 pts) 4. 11

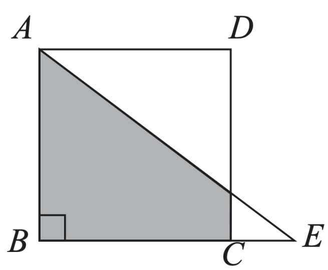
5. What is the smallest positive integer x such that x^2 has 2024 as a factor?

(3 pts) 5. 1012

6. The arithmetic mean A, of any two positive numbers x and y is defined to be $A = \frac{1}{2}(x + y)$ and their geometric mean, G is defined to be $G = \sqrt{xy}$. For two particular values x and y, with $x > y$, the ratio $A : G = 5 : 4$. For these values of x and y, what is the ratio $x : y$?

(3 pts) 6. 4 : 1

7. The diagram shows a square $ABCD$ and a right-angled triangle ABE. The length of BC is 4 and the length of BE is 5. What is the area of the shaded region?



(3 pts) 7. $\frac{48}{5}$ or 9.6

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #4
Grades 11/12

School ______________

Team Name ______________

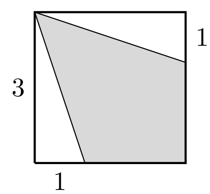
CALCULATORS ARE NOT ALLOWED.

Student Name ______________

1. A box contains 50 balls numbered 1 through 50. Jake draws a ball at random, then Sally draws a ball at random from the remaining 49. If the product of the numbers they draw is even, Sally wins. If the product is odd, Jake wins. Which of the following is true?
   (a) Jake is more likely to win.
   (b) Jake and Sally are equally likely to win.
   (c) Sally is more likely to win.

(2 pts)1. __________

2. What is the area of the shaded region inside the square?



(3 pts)2. __________

3. If a is a positive number and $x^4 + 16 = (x^2 + ax + 4)(x^2 - ax + 4)$, find a.

(3 pts)3. __________

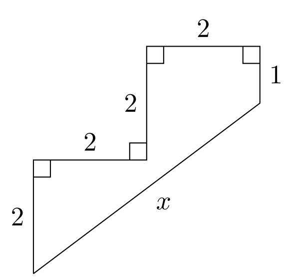
4. Find x if x is positive and: $x = \dfrac{1}{\frac{1}{x+3} + \frac{1}{x+12}}$

(3 pts)4. __________

5. Compute: $\sqrt{24^2 + 25 + 24}$

(3 pts)5. __________

6. Find the length of side x given the information in the picture.



(3 pts)6. __________

7. Compute: $190^2 - 189^2$

(3 pts)7. __________

Total Points __________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #4
Grades 11/12

School ______________

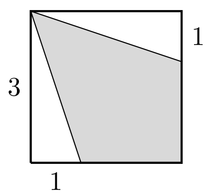
Name ______________

Calculators are NOT allowed.

1. A box contains 50 balls numbered 1 through 50. Jake draws a ball at random, then Sally draws a ball at random from the remaining 49. If the product of the numbers they draw is even, Sally wins. If the product is odd, Jake wins. Which of the following is true?
   (a) Jake is more likely to win.
   (b) Jake and Sally are equally likely to win.
   (c) Sally is more likely to win.

(2 pts)1. c

2. What is the area of the shaded region inside the square?



(3 pts)2. 6

3. If a is a positive number and $x^4 + 16 = (x^2 + ax + 4)(x^2 - ax + 4)$, find a.

(3 pts)3. $2\sqrt{2}$ or $\sqrt{8}$

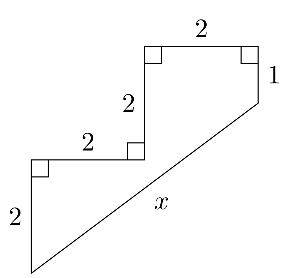
4. Find x if x is positive and: $x = \dfrac{1}{\frac{1}{x+3} + \frac{1}{x+12}}$

(3 pts)4. 6

5. Compute: $\sqrt{24^2 + 25 + 24}$

(3 pts)5. 25

6. Find the length of side x given the information in the picture.



(3 pts)6. 5

7. Compute: $190^2 - 189^2$

(3 pts)7. 379

Total Points ______________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

INDIVIDUAL TEST #4
Grades 11/12

School ______________ Name ______________

CALCULATORS ARE NOT ALLOWED.

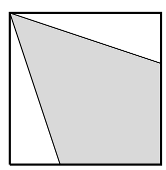
1. A box contains 50 balls numbered 1 through 50. Jake draws a ball at random, then Sally draws a ball at random from the remaining 49. If the product of the numbers they draw is even, Sally wins. If the product is odd, Jake wins. Which of the following is true?

   (a) Jake is more likely to win.
   (b) Jake and Sally are equally likely to win.
   (c) Sally is more likely to win.

   Jake only wins if both numbers are odd. The chances of that are:

$$\frac{1}{2}\left(\frac{24}{49}\right) = \frac{24}{98} < \frac{1}{2}$$

2. What is the area of the shaded region inside the square?



   The shaded area is: $9 - 2\left(\frac{1}{2}(3)(1)\right) = 6$

3. If a is a positive number and $x^4 + 16 = (x^2 + ax + 4)(x^2 - ax + 4)$, find a.

   Note that:

$$\begin{aligned} x^4 + 16 &= (x^2 + ax + 4)(x^2 - ax + 4) \\ &= x^4 - ax^3 + 4x^2 + ax^3 - a^2x^2 + 4ax + 4x^2 - 4ax + 16 \\ &= x^4 + (8 - a^2)x^2 + 16 \end{aligned}$$

   So we need $a > 0$ *with* $a^2 = 8$, *hence* $a = 2\sqrt{2}$.

4. Find x if x is positive and: $x = \dfrac{1}{\frac{1}{x+3} + \frac{1}{x+12}}$

$$x = \frac{(x+12)(x+3)}{x+3+x+12} = \frac{x^2+15x+36}{2x+15}$$

$$2x^2 + 15x = x^2 + 15x + 36$$

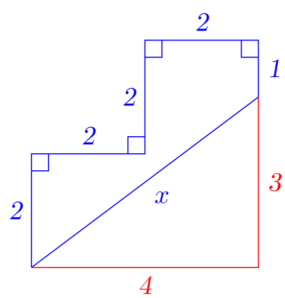
$$x^2 = 36$$

   Since x is positive, $x = 6$.

5. Compute: $\sqrt{24^2 + 25 + 24}$

$\sqrt{24^2 + 25 + 24} = \sqrt{24(24+1) + 25} = \sqrt{24(25) + 25} = \sqrt{25(1+24)} = \sqrt{25^2} = 25$

6. Find the length of side x given the information in the picture.



Using the Pythagorean Theorem we have:

$$x = \sqrt{4^2 + 3^2} = 5$$

7. Compute: $190^2 - 189^2$

$$190^2 - 189^2 = (190 + 189)(190 - 189) = 379$$

Total Points ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

TEAM TEST #1
Grades 11/12

School ______________

Team Name ______________

Calculators are allowed.

1. A rectangular piece of plywood is 3.6 feet wide and 7.2 feet long. Find the area of the piece of plywood. Round your answer to one decimal place. Express your answer in square feet.

(20 pts) 1. ________

2. A used car dealer has six cars to sell. Two of the cars are red, and the other four cars are white. The dealer wishes to park all six of the cars in a row in front of the showroom building. The row of cars will be along the street, and all six cars will face the street. The dealer has enough space to park six cars along the street, but no more than six cars. The two red cars must be parked adjacent to each other. How many arrangements of the six cars are possible?

(20 pts) 2. ________

3. A large bowl contains 22 balls. All of the balls are of the same size and shape, and all are made of the same material. Of the 22 balls, 9 are red, 7 are white, and 6 are blue. Suppose that we randomly select two of the 22 balls and place both of the selected balls together in a different bowl. What is the probability that neither of the selected balls is red?

(20 pts) 3. ________

4. Mrs. Noriega is designing a box. The box is to have six sides. Each of the six sides is to be a rectangle. The dimensions of the box, in inches, are to be x by $12 - 2x$ by $9 - 2x$. Here x is a number which Mrs. Noriega has not yet determined. Find the value of x which will result in the box with the largest possible volume. Round your answer to two decimal places. Hint: All three of the dimensions of the box must be positive real numbers.

(20 pts) 4. ________

5. The cost of a ticket for a certain baseball game was \$25. Children under the age of 18 received a discount, however. The cost of a "youth" ticket was \$17. The team sold 748 tickets for the game. The total sales revenue for these 748 tickets was \$17,532. How many youth tickets did the team sell?

(20 pts) 5. ________

6. What is the area of the region in the xy-plane whose points (x, y) satisfy the following inequality?

$$|x| + |y| + |x + y| \leq 2$$

(20 pts) 6. ________

7. Mrs. Ramaphosa drives ten miles to work each day. She leaves for work at the same time every morning and arrives at work at exactly 8:00 A.M. Her average travel speed each morning is 40 miles per hour. One morning, however, the traffic is congested. After Mrs. Ramaphosa has traveled exactly two miles, she determines that her average speed up to that point has been only 24 miles per hour. If Mrs. Ramaphosa is to arrive at work at exactly 8:00 A.M., what must her average speed for the remaining 8 miles of the trip be? Hint: Find the required average speed for the last 8 miles of the trip only.

(20 pts) 7. ________

8. Suppose that two sides of an isosceles triangle have length 3, and let x equal the length of the third side of the triangle. What value of x will produce a triangle with the largest possible area? Round your answer to two decimal places.

(20 pts) 8. ________

9. A *disk* is a circle together with the points inside the circle. Suppose that disks of equal radii are arranged in a regular pattern throughout the plane in such a way that each disk touches and is tangent to six other disks. What percentage of the plane is covered by the disks? Round your answer to two decimal places. For example, your answer might be 72.34%, 47.83%, or something similar to these percentages.

(20 pts) 9. ________

10. Four solid spheres lie on the top of a table. Each sphere is tangent to each of the other three spheres. Three of the four spheres have radius 4 cm, and the other sphere has a radius of less than 4 cm. What is the radius of the smaller sphere? Round your answer to two decimal places.

(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

Key

TEAM TEST #1
Grades 11/12

School ______________

Team Name ______________

Calculators are allowed.

1. A rectangular piece of plywood is 3.6 feet wide and 7.2 feet long. Find the area of the piece of plywood. Round your answer to one decimal place. Express your answer in square feet.

(20 pts) 1. 25.9 ft.^{2}

2. A used car dealer has six cars to sell. Two of the cars are red, and the other four cars are white. The dealer wishes to park all six of the cars in a row in front of the showroom building. The row of cars will be along the street, and all six cars will face the street. The dealer has enough space to park six cars along the street, but no more than six cars. The two red cars must be parked adjacent to each other. How many arrangements of the six cars are possible?

240 or 240 arrangements

(20 pts) 2. ______

3. A large bowl contains 22 balls. All of the balls are of the same size and shape, and all are made of the same material. Of the 22 balls, 9 are red, 7 are white, and 6 are blue. Suppose that we randomly select two of the 22 balls and place both of the selected balls together in a different bowl. What is the probability that neither of the selected balls is red?

0.338 or 0.34 or 33.8% or 34%, or $\frac{26}{77}$, but not 0.3 or $\frac{3}{10}$

(20 pts) 3. ______

4. Mrs. Noriega is designing a box. The box is to have six sides. Each of the six sides is to be a rectangle. The dimensions of the box, in inches, are to be x by $12 - 2x$ by $9 - 2x$. Here x is a number which Mrs. Noriega has not yet determined. Find the value of x which will result in the box with the largest possible volume. Round your answer to two decimal places. Hint: All three of the dimensions of the box must be positive real numbers.

1.70 inches or 1.70

(20 pts) 4. ______

5. The cost of a ticket for a certain baseball game was \$25. Children under the age of 18 received a discount, however. The cost of a "youth" ticket was \$17. The team sold 748 tickets for the game. The total sales revenue for these 748 tickets was \$17,532. How many youth tickets did the team sell?

(20 pts) 5. 146 or 146 youth tickets

6. What is the area of the region in the xy-plane whose points (x, y) satisfy the following inequality?

$$|x| + |y| + |x + y| \leq 2$$

(20 pts) 6. 3

7. Mrs. Ramaphosa drives ten miles to work each day. She leaves for work at the same time every morning and arrives at work at exactly 8:00 A.M. Her average travel speed each morning is 40 miles per hour. One morning, however, the traffic is congested. After Mrs. Ramaphosa has traveled exactly two miles, she determines that her average speed up to that point has been only 24 miles per hour. If Mrs. Ramaphosa is to arrive at work at exactly 8:00 A.M., what must her average speed for the remaining 8 miles of the trip be? Hint: Find the required average speed for the last 8 miles of the trip only.

(20 pts) 7. 48 miles per hour

8. Suppose that two sides of an isosceles triangle have length 3, and let x equal the length of the third side of the triangle. What value of x will produce a triangle with the largest possible area? Round your answer to two decimal places.

(20 pts) 8. 4.24

9. A *disk* is a circle together with the points inside the circle. Suppose that disks of equal radii are arranged in a regular pattern throughout the plane in such a way that each disk touches and is tangent to six other disks. What percentage of the plane is covered by the disks? Round your answer to two decimal places. For example, your answer might be 72.34%, 47.83%, or something similar to these percentages.

(20 pts) 9. 90.69 %

10. Four solid spheres lie on the top of a table. Each sphere is tangent to each of the other three spheres. Three of the four spheres have radius 4 cm, and the other sphere has a radius of less than 4 cm. What is the radius of the smaller sphere? Round your answer to two decimal places.

(20 pts) 10. 1.33 cm

TOTAL POINTS ________

2024–2025 Math Track Meet
Team Test #1, Grades 11/12
Calculators are allowed.

1. (3.6) times (7.2) = 25.9 ft.^{2}

2. Think of the six parking places along the street. We may number these parking places from 1 to 6.

___ ___ ___ ___ ___ ___
1 2 3 4 5 6

The left most of the red cars can be in spot 1, 2, 3, 4, or 5.

There are five ways to choose the spot for the leftmost red car. There are two ways of arranging the two red cars in this spot and the spot immediately to the right of this spot. There are $4 \cdot 3 \cdot 2 \cdot 1 = 24$ ways to arrange the four white cars in the four slots not occupied by red cars.

The number of ways of arranging the six cars in the six slots is thus

$5 \times 2 \times 24 = 240.$

There are thus 240 possible arrangements

2024 - 2025 Math Track Meet

Team Test #1, Grades 11/12

Calculators **are** allowed.

3. Imagine that we select the two balls one at a time.

13 of the 22 balls are of a color other than red.

The probability that the first ball is **not** red is

$$\frac{13}{22}.$$

If the first ball is not red, consider the situation just before we randomly select the second ball. There are 21 remaining balls, and 12 of them are of a color other than red. The probability that the second ball is not red, given that the first ball was not red is $\frac{12}{21}$.

The probability that neither of the first two balls is red is

$$\frac{13}{22} \times \frac{12}{21} = \frac{13 \cdot 6}{11 \cdot 21} = \frac{13 \cdot 2}{11 \cdot 7} = \frac{26}{77} = \boxed{0.338}$$

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Team Test #1, Grades 11/12

Calculators <u>are</u> allowed.

4. The dimensions of the box are to be positive numbers, so we require

$x > 0$, $12 - 2x > 0$, and $9 - 2x > 0$.

Thus $12 > 2x$, or $6 > x$.
Similarly, $9 > 2x$, or $x < 4.5$.

So $0 < x < 4.5$.

The volume of the box will be

$V(x) = x(12-2x)(9-2x)$.

We draw the graph of $V(x)$ on a graphing calculator. Our initial viewing window is $[0, 4.5]$ by $[0, 100]$.
By zooming in on the graph, we find that the maximum value of V occurs when $x = 1.70$

TS

2024-2025 Math Track Meet

Team Test #1, Grades 11/12

Calculators are allowed.

5. Let x = the number of adult tickets sold.

Let y = the number of youth tickets sold.

Then

(1) $x + y = 748$

(2) $25x + 17y = 17{,}532$

By (1), $x = 748 - y$.

We plug this into (2). We find that

$25(748 - y) + 17y = 17{,}532$

$18700 - 25y + 17y = 17{,}532$

$18700 - 8y = 17{,}532$

$18700 - 17532 = 8y$

$1168 = 8y$

$146 = y$

$8 \overline{)1168}$ = 146

146 youth tickets were sold.

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Team Test #1, Grades 11/12

Calculators are allowed

6. We consider points in the four quadrants separately.

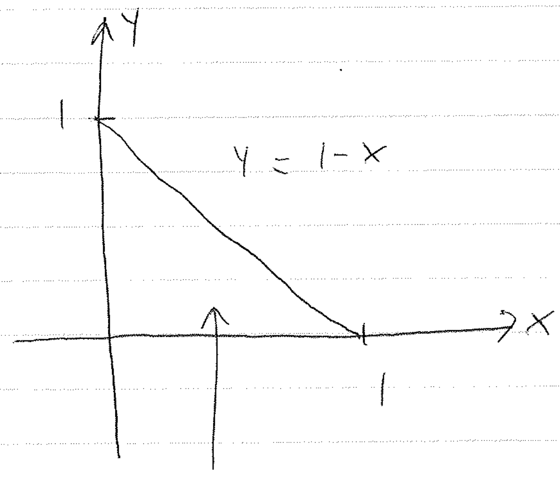
Quadrant I

In this quadrant, $x \geq 0$ and $y \geq 0$. So $|x| = x$, $|y| = y$, and $|x+y| = x+y$.

$|x| + |y| + |x+y| \leq 2$ if and only if

$x + y + x + y \leq 2$, i.e. $2x + 2y \leq 2$.

This is the same as $x + y \leq 1$, or $y \leq 1 - x$



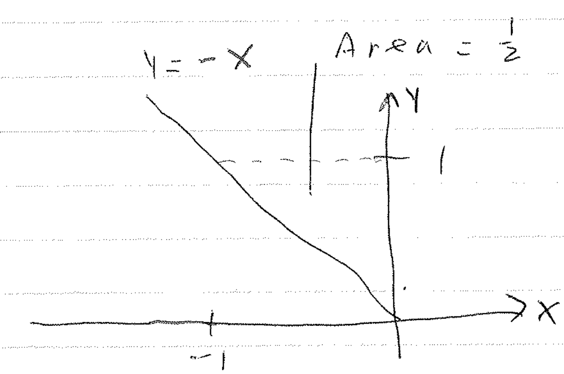
Area = $\frac{1}{2}$

Quadrant II

Case 1: $y \geq -x$

$x + y \geq 0$

$x \leq 0$, $y \geq 0$



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Team Test #1, Grades 11/12

Calculators are allowed

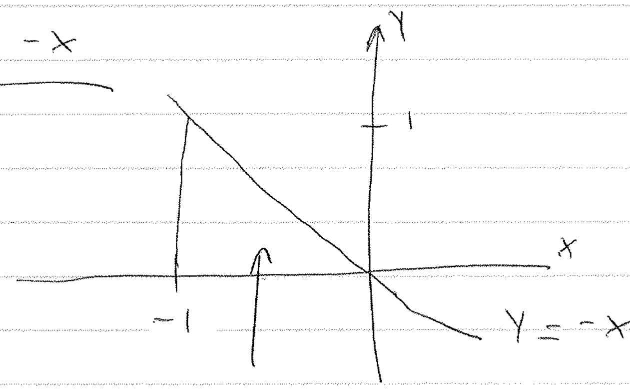
#6, continued

$|x| + |y| + |x+y| \le 2$ if and only if

$-x + y + x + y \le 2$, or $2y \le 2$, i.e. $y \le 1$.

Area $= \frac{1}{2}$. See diagram on previous page.

Case 2: $y \le -x$



$x + y \le 0$,

$|x+y| = -x-y$

$|x| = -x$

$|y| = y$

Area $= \frac{1}{2}$

$|x| + |y| + |x+y| \le 2$ if and only if

$-x + y - x - y \le 2$, i.e. $-2x \le 2$, or

$x \ge -1$. Area $= \frac{1}{2}$.

Total area for Quadrant II: 1

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Team Test #1, Grades 11/12

Calculators are allowed.

#6, continued

Quadrant III

$x \le 0,\ y \le 0,\quad x + y \le 0.$

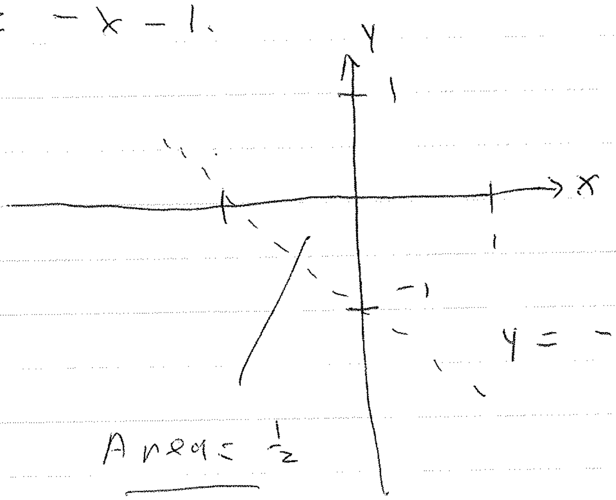
$|x| = -x,\ |y| = -y,\quad |x+y| = -x-y.$

$|x| + |y| + |x+y| \le 2$ if and only if

$-x - y - x - y \le 2$, i.e. $-2x - 2y \le 2$.

This is the same as $x + y \ge -1$, i.e.

$y \ge -x - 1.$

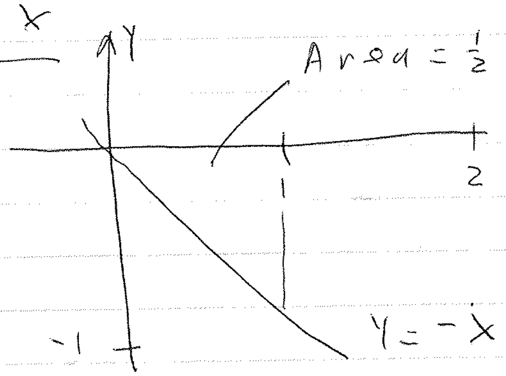


Area = $\frac{1}{2}$

Quadrant IV

case 1: $y \ge -x$

$x + y \ge 0$
$x \ge 0$
$y \le 0$



#6, continued

$|x| + |y| + |x+y| \le 2$ if and only if

$x - y + x + y \le 2$, i.e. $2x \le 2$, or $x \le 1$.

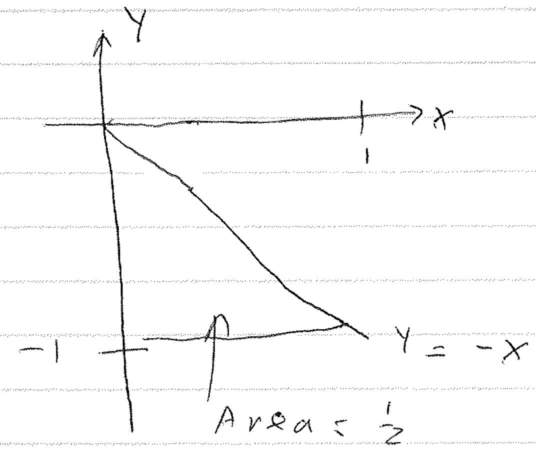
Area $= \frac{1}{2}$. See diagram on previous page.

Case 2: $y \le -x$

$x + y \le 0$

$|x+y| = -x - y$

$x \ge 0$
$y \le 0$



$|x| + |y| + |x+y| \le 2$ if and only if

$x - y - x - y \le 2$, i.e. $-2y \le 2$.

Thus $y \ge -1$.

Area $= \frac{1}{2}$

Total area for Quadrant IV: 1

If we consider our results for the four quadrants, we find that the total area of the original region is $\frac{1}{2} + 1 + \frac{1}{2} + 1 = \boxed{3}$.

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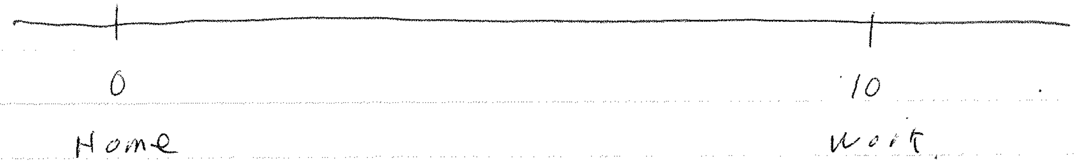
Team Test #1, Grades 11/12
Calculators are allowed.

7.



let t = the time, in hours, that it takes Mrs. Ramaphosa to drive to work if her average speed is 40 mph, Then

$$\left(40 \frac{\text{miles}}{\text{hour}}\right) t \text{ hours} = 10 \text{ miles}$$

$$t = \frac{10 \text{ miles}}{40 \text{ miles/hour}} = \frac{1}{4} \text{ hour.}$$

So Mrs. Ramaphosa leaves for work at 7:45 A.M. each Day. For the morning in question, let s = the time, in hours, that it takes for Mrs. Ramaphosa to travel the first two miles of her trip to work. Then

$$\left(24 \frac{\text{miles}}{\text{hour}}\right) s \text{ hours} = 2 \text{ miles.}$$

$$\text{So } s = \frac{2 \text{ miles}}{24 \text{ miles/hour}} = \frac{1}{12} \text{ hours.}$$

For the morning in question, consider the situation after Mrs. Ramaphosa has traveled exactly two miles. She left for work at 7:45 A.M., and she has traveled for $\frac{1}{12}$ of an hour.

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Team Test # 1, Grades 11/12

calculators are allowed.

#7, continued. But $\frac{1}{12}$ hours $= \left(\frac{1}{12}\text{ hours}\right)\frac{60\text{ minutes}}{\text{hour}} =$ 5 minutes.

If Mrs. Ramaphosa is to arrive at work at exactly 8:00 A.M., she must cover the remaining 8 miles of her trip to work in exactly 10 minutes.

$$10\text{ minutes} = (10\text{ minutes})\frac{1\text{ hour}}{60\text{ minutes}} = \frac{1}{6}\text{ hours}.$$

Her average speed for the remainder of the trip must be

$$\frac{8\text{ miles}}{\frac{1}{6}\text{ hours}} = 8 \cdot \frac{6}{1}\,\frac{\text{miles}}{\text{hour}} =$$

48 miles per hour.

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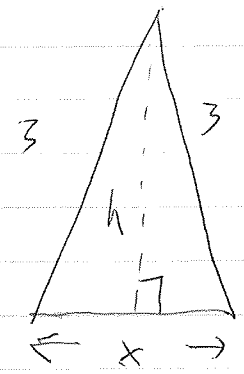
Team Test #1, Grades 11/12
Calculators are allowed.

8.



By the Pythagorean Theorem,

$$h = \sqrt{9 - (\tfrac{1}{2}x)^2} = \sqrt{9 - x^2/4}$$

Let A = the area of the isosceles triangle. Then

$$A = \tfrac{1}{2}bh = \tfrac{1}{2}x\sqrt{9 - \tfrac{x^2}{4}}$$

To find the value of x that maximizes A, we may find the value of x which maximizes A^2. But

$$A^2 = \tfrac{1}{4}x^2\left(9 - \tfrac{x^2}{4}\right).$$

This choice of x also maximizes $16A^2$, i.e.

$$x^2(36 - x^2).$$

Let $f(x) = x^2(36 - x^2)$. Then

$$f(x) = -x^2(x^2 - 36) = -(x^4 - 36x^2)$$

$$= -(x^4 - 36x^2 + 18^2) + 18^2$$

$$= -(x^2 - 18)^2 + 18^2$$

But $y = -(w - 18)^2 + 18^2$ is a

2024-2025 Math Track Meet
Team Test #1, Grades 11/12
Calculators are allowed.

#8, continued. parabola which opens downward. The maximum value of y occurs when $u = 18$. Thus $f(x)$ is maximized when $x^2 = 18$, i.e. $x = 3\sqrt{2}$ $= 4.24$. The desired value of x is thus 4.24.

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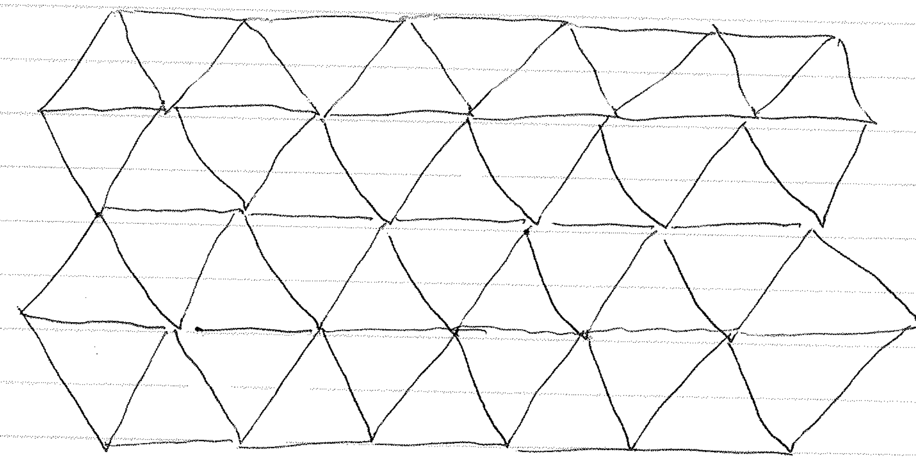
Team Test #1, Grades 11/12

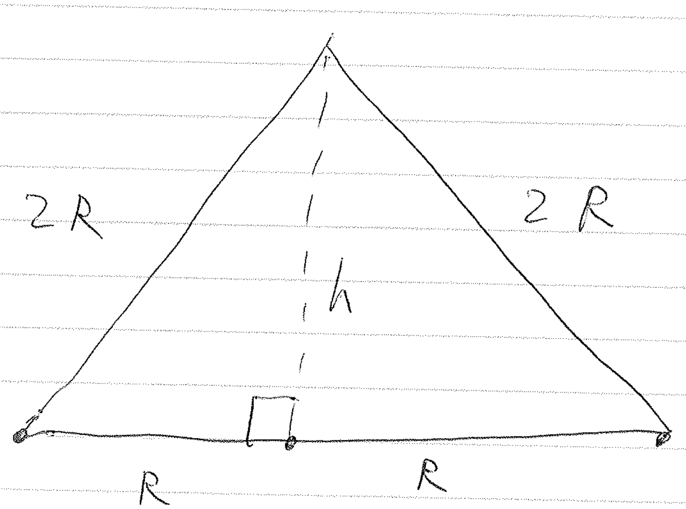
Calculators are allowed.

9. Let R denote the radius of each disk. We may draw equilateral triangles throughout the plane as follows.

(*)



Suppose that each side of each of these equilateral triangles has length $2R$. Consider one such equilateral triangle.



$(2R)^2 = R^2 + h^2$, so

$$h^2 = 4R^2 - R^2 = 3R^2$$

$$h = \sqrt{3}R$$

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Calculators are allowed.

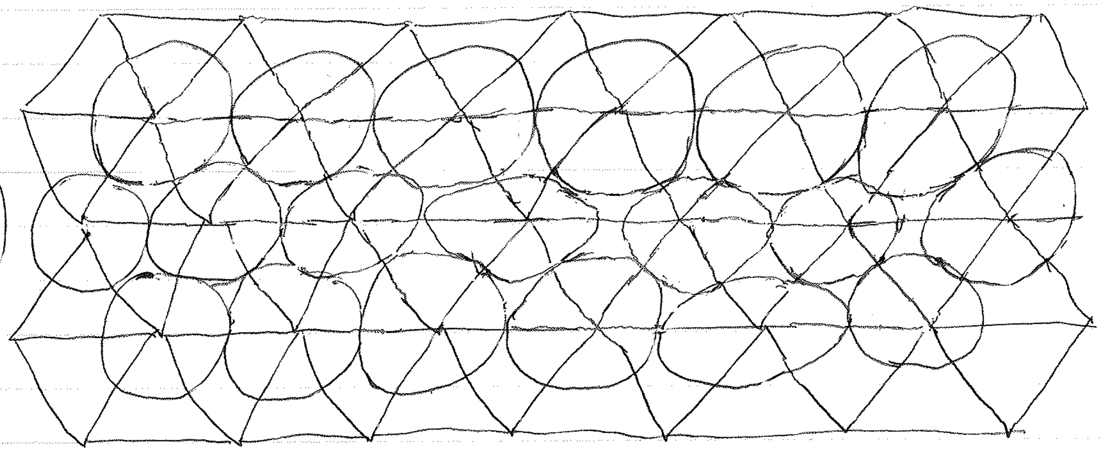
#9, continued

So the area of each equilateral triangle is

$$\frac{1}{2} b h = \frac{1}{2} 2R \sqrt{3} R = \sqrt{3} R^2$$

In (*), above, draw disks of radius R centered at the vertices of all of the equilateral triangles.

(**)



If one draws (**) more carefully, each disk will touch and be tangent to six other disks. The area of each disk will be πR^2.

Each equilateral triangle in (**) contains sectors of 3 disks. Each sector has area $\frac{\pi R^2}{6}$. The total area of these three sectors is $\frac{\pi R^2}{2}$.

The fraction of each equilateral triangle covered by the sectors is

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Team Test #1, Grades 11/12
Calculators are allowed.

#9, continued

$$\frac{\frac{\pi R^2}{2}}{\sqrt{3}\, R^2} = \frac{\pi R^2}{2} \cdot \frac{1}{\sqrt{3} R^2} = \frac{\pi}{2\sqrt{3}} =$$

$\frac{\pi \sqrt{3}}{6}$. The fraction of the entire plane covered by the disks is thus

$$\frac{\pi \sqrt{3}}{6} \times 100 = 90.69\,\%$$

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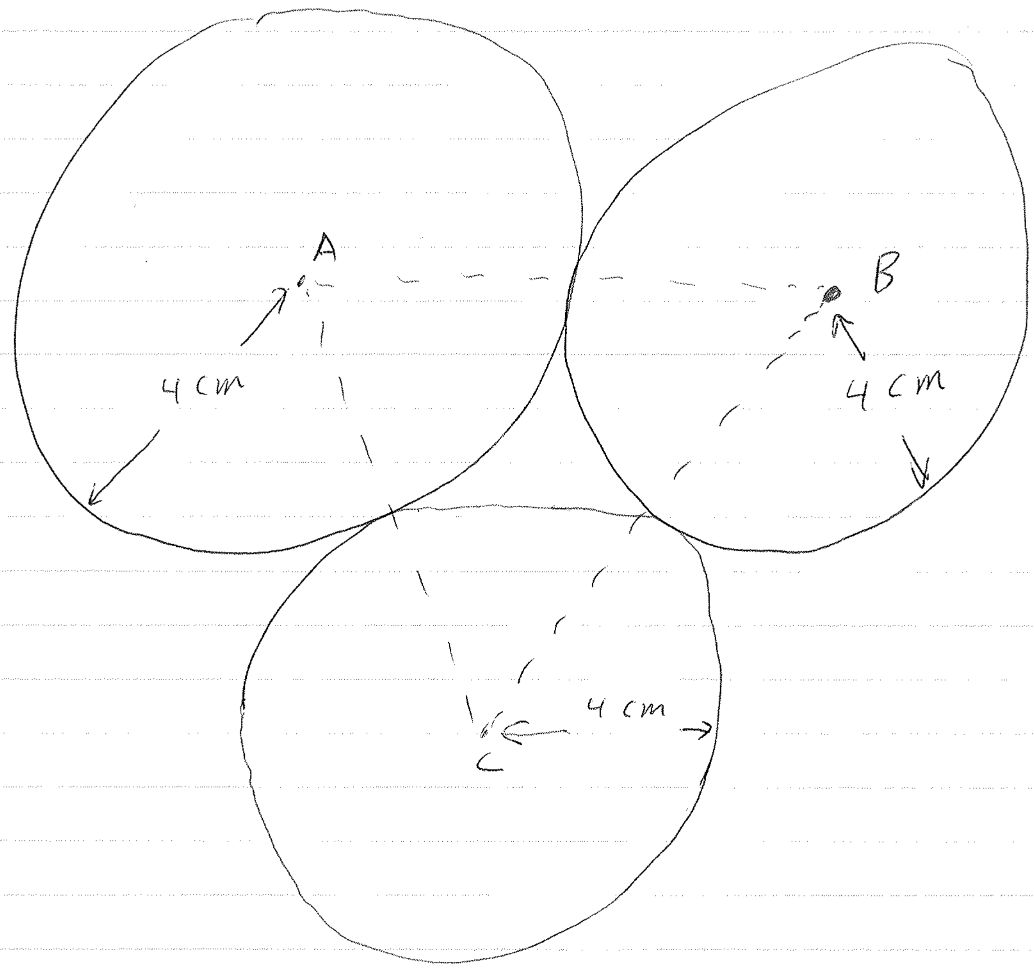
Team Test # 1, Grades 11/12

Calculators <u>are</u> allowed.

10. Consider the three spheres of radius 4 cm. If we project these spheres onto the table, we obtain the following result:



The points A, B, and C are the centers of the circles shown. These points are the vertices of an equilateral triangle. Each side of this equilateral triangle has length 8 cm. Consider this triangle.

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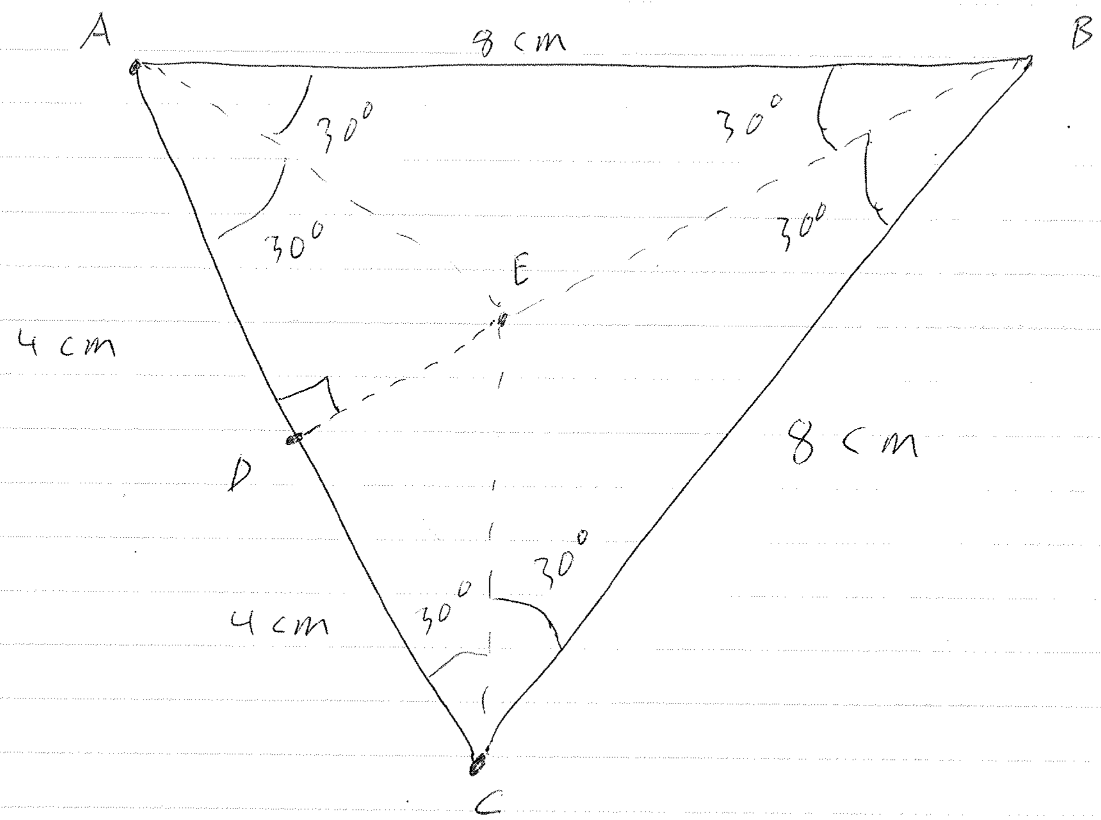
Team Test #1, Grades 11/12

Calculators are allowed

#10, continued



$\cos 30° = \frac{|AD|}{|AE|}$. Thus $|AE| = \frac{|AD|}{\cos 30°}$

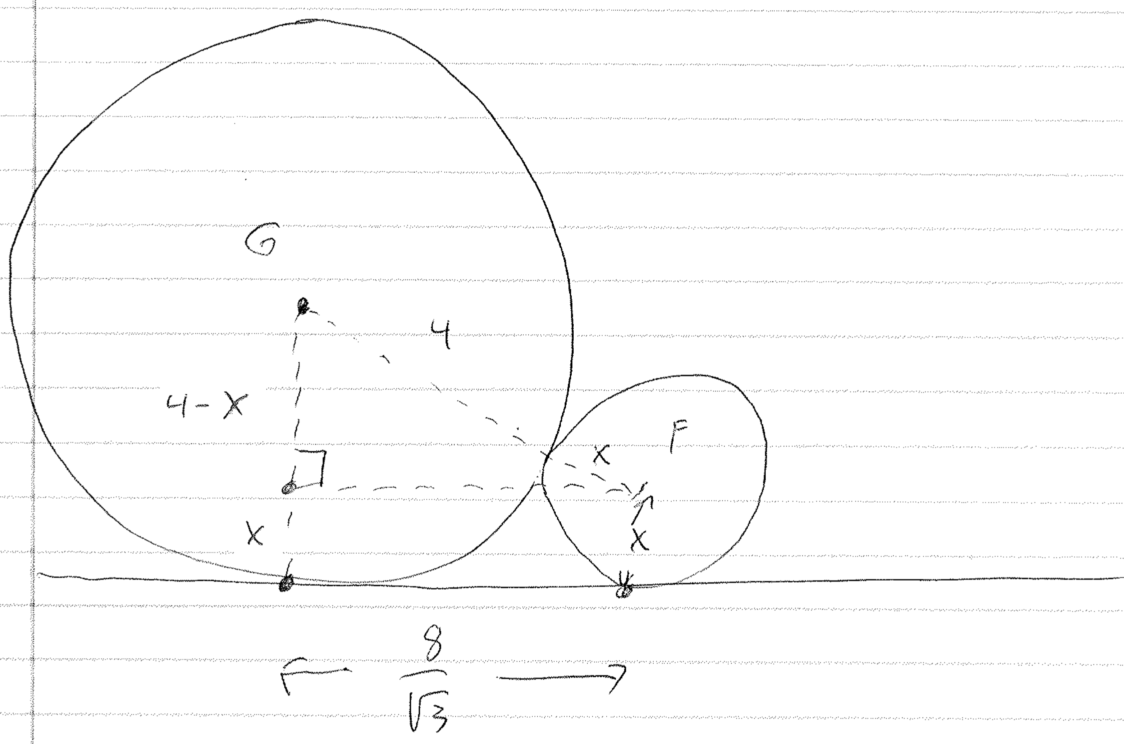
$= \frac{4}{\frac{\sqrt{3}}{2}} = 4 \cdot \frac{2}{\sqrt{3}} = \frac{8}{\sqrt{3}}$

$|AE| = \frac{8}{\sqrt{3}}$

The center of the smaller sphere lies directly above E. Let F be the center of the smaller sphere. Let G be the center of any of the larger

2024 - 2025 Math Track Meet
Team Test #1, Grades 11/12
Calculators <u>are</u> allowed

#10, continued. Spheres. Let x denote the radius of the smaller sphere. Let P denote the plane perpendicular to the table which contains F and G. The intersection of P with the spheres centered at F and G is as follows:



By the Pythagorean Theorem,

$$(4+x)^2 = (4-x)^2 + \left(\frac{8}{\sqrt{3}}\right)^2.$$ Thus

$$4^2 + 8x + x^2 = 4^2 - 8x + x^2 + \frac{64}{3}$$

$$16x = \frac{64}{3}, \quad x = \frac{4}{3} = 1.33$$

The radius of the smaller sphere 1.33 cm.

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

TEAM TEST #2
Grades 11/12

School ______________ Team Name ______________

Calculators are **NOT** allowed.

1. Recall $i^2 = -1$. Simplify $(1+i)^8$ as much as possible.

(20 pts) 1. ________

2. What is the average of the three solutions to $x^3 + 6x^2 - 8x - 16$?

(20 pts) 2. ________

3. Find all positive values for a radius of a circle for which the area of the circle is equal numerically to twice its circumference.

(20 pts) 3. ________

4. Find all solutions to $x - 1 = \sqrt{2x+1}$.

(20 pts) 4. ________

5. Assume today is Friday. What day of the week will it be in 365 days?

(20 pts) 5. ________

6. How many edges does a tetrahedron have?

(20 pts) 6. ________

7. How many real roots does the equation $\cos x = \log_{10} x$ have?

(20 pts) 7. ________

8. Sort the integers 1 through 4 randomly, and denote them as a, b, c, and d. What is the probability that $ab + cd$ is an even number?

(20 pts) 8. ________

9. A penny is placed flat on a table. What is the maximum number of pennies that can be places around it, flat on the table, with each one tangent to it?

(20 pts) 9. ________

10. In a group of dogs and people, the number of legs was 14 more than twice the number of heads. How many dogs were there. [Assume none of the people or dogs is missing a leg or has an extra.].

(20 pts) 10. ________

TOTAL POINTS ________

UND MATHEMATICS TRACK MEET
University of North Dakota
January 13, 2025

TEAM TEST #2
Grades 11/12

School ______________ **Key** Team Name ______________

Calculators are **NOT** allowed.

1. Recall $i^2 = -1$. Simplify $(1+i)^8$ as much as possible.

   (20 pts) 1. 16

2. What is the average of the three solutions to $x^3 + 6x^2 - 8x - 16$?

   (20 pts) 2. -2

3. Find all positive values for a radius of a circle for which the area of the circle is equal numerically to twice its circumference.

   (20 pts) 3. 4

4. Find all solutions to $x - 1 = \sqrt{2x+1}$.

   (20 pts) 4. $x = 4$

5. Assume today is Friday. What day of the week will it be in 365 days?

   (20 pts) 5. Saturday

6. How many edges does a tetrahedron have?

   (20 pts) 6. 6

7. How many real roots does the equation $\cos x = \log_{10} x$ have?

   (20 pts) 7. 3

8. Sort the integers 1 through 4 randomly, and denote them as a, b, c, and d. What is the probability that $ab + cd$ is an even number?

   (20 pts) 8. $\frac{2}{3}$

9. A penny is placed flat on a table. What is the maximum number of pennies that can be places around it, flat on the table, with each one tangent to it?

   (20 pts) 9. 6

10. In a group of dogs and people, the number of legs was 14 more than twice the number of heads. How many dogs were there. [Assume none of the people or dogs is missing a leg or has an extra.].

    (20 pts) 10. 7

Math Track Meet 2025
Team Test #2
Grades 11/12
Solutions

1. Observe $(1+i)^2 = 1 + 2i + i^2 = 2i$ and so $(1+i)^8 = \left((1+i)^2\right)^4 = (2i)^4 = 16i^4 = 16$.

2. Denote the three solutions to $x^3 + 6x^2 - 8x - 16$ by r_1, r_2, and r_3. By Viete's formula, $r_1 + r_2 + r_3 = -6$. Thus $\frac{r_1+r_2+r_3}{3} = -\frac{6}{3} = -2$.

3. Let r be the radius of a circle. The condition $A = 2C$ for the area A and circumference C can be expressed in terms of r as $\pi r^2 = 2(2\pi r)$. Rearranging gives $\pi r^2 - 4\pi r = 0$ or equivalently $\pi r(r-4) = 0$. The only positive solution to the latter is $r = 4$.

4. Observe

$$\begin{aligned} x - 1 &= \sqrt{2x+1} \\ (x-1)^2 &= 2x+1 \\ x^2 - 2x + 1 &= 2x + 1 \\ x^2 - 4x &= 0 \\ x(x-4) &= 0 \end{aligned}$$

which has solutions $x = 0$ and $x = 4$. Checking each in the original equation, we verify that $x = 4$ is a solution and that $x = 0$ is not a solution. Thus $x = 4$ is the only solution.

5. Observe $365 = 52(7) + 1$, so 365 days after Fri. is the same as 1 day after Fri. This is Sat.

6. A tetrahedron is a platonic solid with four triangular faces and 6 edges.

7. Observe that $|\cos x| \leq 1$ and $\log_{10} x > 1$ for $x > 10$. It follows that any solutions occur in the interval $(0, 10)$. Now $\log_{10} x \leq 0$ for $0 < x \leq 1$ and $\cos x > 0$ on $(0, \pi/2)$. Thus there are no solutions in the interval $(0, 1]$. On $(1, 10]$, we have $0 < \log_{10} x \leq 1$. There is one solution in the interval $(1, \pi/2)$ and two more solutions in the interval $(3\pi/2, 5\pi/2)$. Noting that $7\pi/2 > 10$, there are no additional solutions. In summary, there are three solutions, namely $x \approx 1.418, 5.552, 6.863$ to three digit places.

8. There are $4!$ possible ways to assign values to a, b, c, and d. We count the ways for which $ab+cd$ is odd. This requires that ab or cd is odd. Now ab is odd in $2!\cdot 2! = 4$ ways. Similarly, cd is odd in $2!\cdot 2! = 4$ ways. Thus there are 8 ways for $ab+cd$ to be odd. So, there are $24-8 = 16$ ways for $ab+cd$ to be even. We conclude that the probability that $ab+cd$ is an even number is $\frac{16}{24} = \frac{2}{3}$.

9. When two pennies are placed around the original penny, the line segments between the centers of the three coins form an equilateral triangle. Thus the central angle formed between the two placed is 60 degrees with respect to the center penny. It follows that the number of pennies which can be placed is $360/60 = 6$.

10. Let d and p be the number of dogs and people respectively. Then $4d + 2p = 2(d+p) + 14$. Thus $2d = 14$ and $d = 7$. There are 7 dogs.