7/8 problems

1. Compute the remainder when 3325 is divided by 97.

3325 = 34*97+27 so the answer is 27.

2. George drinks [4/3] cups of water each weekday and drinks [12/7] cups of water one each of Saturday and Sunday. How many cups of water does George drink each week?

$5*{}^{4}/_{3}+2*[12/7] = [(140+72)/21] = 10[2/21]$ cups

3. Two robins just finished eating the berries off of a bush. The first flies away East at 8 miles per hour. The second flies North at 6 miles per hour. How many minutes will it take for the birds to be 25 miles apart?

Let t be the number of minutes. Then $(8[t/60])^2+(6[t/60])^2 = 25^2$. So $(8t)^2+(6t)^2 = 25^2*60^2$. That is $100t^2 = 100*25*60*15$ so $t^2 = 5^2*15^2*2^2$. Thus t = 150 minutes.

4. The prime factorization of 374 is:

374 = 2*11*17

5. Compute $5^3+4^2+3^1+2^0$.

$5^3+4^2+3^1+2^0 = 125+16+3+1 = 145$

6. Solve the inequality $|x–4| < 6$.

We have $–6 < x–4 < 6$, so $–2 < x < 10$.

7. The hands of a clock are positioned at 3:42 with the hour hand pointed directly at the 3. How many degrees separate the hands in the smaller section?

The angle measure between two consecutive minute marks on the clock is 6 degrees. So the hands are separated by (42–15)*6 = 27*6 = 162 degrees.

8. A rectangular solid has width that is twice its height and length that is five times its height. Find the total surface area of the solid in terms of its height.

$A = 2wl+2wh+2hl = 2(2h)(5h)+2(2h)h+2h(5h) = 34h^2$.

9. $12211_3 = \quad\quad\quad _{10}$

$12211_3 = 3^4+2*3^3+2*3^2+3^1+3^0 = 81+54+18+3+1 = 157_{10}$

10. If a car gets 20 miles per gallon and it is traveling at 60 miles per hour, how many gallons per minute is it using?

[60 miles/ hour]*[1 hour/60 minutes]*[1 gallon/20 miles] = [1/20] gallons per minute.

9/10 problems

1. Let n be the number of ways that 10 dollars can be changed into dimes and quarters, with at least one of each coin being used. Find n.

If d is the number of dimes and q is the number of quarters used to make change, $10d+25q = 1000$. So $5q = 2(100-d)$. This means that 5 must divide $100-d$. Thus d is a multiple of 5 with $1 \le d < 100$. There are 19 such numbers.

2. If the sum of two numbers is 1, and their product is 1, find the sum of their cubes.

Given $x+y = 1$ and $xy = 1$ we substitute $y = 1/x$ to yield $x+1/x = 1$. Multiplying by x gives $x^2-x+1 = 0$ which has as roots $(1\pm\sqrt{[(-3)]})/2$. These are the primitive sixth roots of one. Each cubed thus equals -1. So the sum of their cubes is -2.

3. A prime number is an integer $p > 1$ such that 1 and p are the only positive factors of p. The digits 1,3,4,6,8, and 9 are each used exactly once to form three two-digit primes. What is the sum of the three prime numbers?

First realize that each of the three primes must be odd. So the number's first digits are one of 4,6 and 8 and their second digits are 1,3 or 9. Since 63 and 69 are not prime, one of our numbers must be 61. Since 49 is not prime, the second is 43, and the third is 89. Their sum is 193.

4. The sum of two numbers is 7, and their product is 25. What is the sum of their reciprocals?

$^1/_x+^1/_y$ = [(y+x)/xy] = [7/25].

5. Lena was traveling from Grand Forks to Minot by bus. When the bus had traveled half the distance, Lena fell asleep. When she awoke her distance to Minot was half the distance the she had traveled while asleep. For what fraction of the trip did Lena sleep?

If x is the distance to Minot when Lena wakes up, then x+2x is half the distance from Grand Forks to Minot. So x is 1/6 of the distance from Grand Forks to Minot. Therefore the answer is 1/3.

6. How many 7-digit numbers contain the digit 7 at least once?

$9\cdot10^6-8\cdot9^6$ = 4,748,472. From all 7-digit numbers, exclude those which never contain the digit 7.

7. Given a square ABCD with M and N the midpoints of sides AB and DC and ∠ NDM = θ, find the exact value of sinθ.

Let E be the midpoint of MN and s be the length of one side of the square. Denote the length of a segment PQ by |PQ|. Both |NB| = |BM| = s/2 by choice of M and N. So |MN| = s√2/2 by applying the Pythagorean theorem to triangle BMN. Thus |NE| = s√2/4. Next |ND| = s√5/2 by applying the Pythagorean theorem to triangle DCN. Finally applying the Pythagorean theorem to triangle END gives |DE| = 3s√2/4. Since E bisects MN, ∠EDN = θ/2. So sinθ/2 = √[10]/10, and cosθ/2 = 3√[10]/10. Thus sinθ = 2sinθ/2 cosθ/2 = 2√[10]/10·3√[10]/10 = 3/5.

8. If $g_1(x) = (x-2)^2$, $g_2(x) = x-3$, $g_3(x) = |x|$, and $g_4(x) = 1/x$, find the value f(1), where $f(x) = g_4(g_3(g_2(g_1(x))))$.

$f(1) = g_4(g_3(g_2(g_1(1)))) = g_4(g_3(g_2(1))) = g_4(g_3(-2)) = g_4(2) = 1/2$.

9. If 3 is the first term in the sequence 3,10,17,24,... which term of the sequence is equal to 129?

The terms satisfy $a_n = 7n-4$. So 129 is the 19th term.

10. What is the value of the sum $2^{-1}+2^{-2}+...+2^{-10}$?

$\frac{1}{2}[(1-\frac{1}{2}^{10})/(1-\frac{1}{2})] = 1-\frac{1}{2}^{10} = 1-[1/1024] = 1023/1024$

11/12 problems

1. What is the remainder when x^6+1 is divided by $x-3$?

If $x^6+1 = q(x)(x-3)+r$, setting $x = 3$ yields $r = 3^6+1 = 730$.

2. Find the center of the circle which passes through (2,0),(6,0) and (6,6).

Say the center is (h,k), then $(2-h)^2 = (6-h)^2$ implies $h = 4$. Next $k^2 = (6-k)^2$ implies $k = 3$.

3. Reduce 2,690,151/9,863,887 to lowest terms.

Run the Euclidean algorithm. 9,863,887=3*2,690,151+1,793,434; 2,690,151=1*1,793,434+896,717;

1,793,434=2*896,717. Since 896,717 goes into the numerator 3 times and the denominator 11 times, the answer is 3/11.

4. A ball of string 2 inches in radius takes 400 feet of string. What length of string is needed to make a ball which is 3 inches in radius?

The amount of string varies cubicly with the radius with constant 50. So one needs 1350 ft of string.

5. If $f(x) = [(x-1)/(x+1)]$ find $f^{-1}(x)$.

$x = [(y-1)/(y+1)]$ means $yx+x = y-1$. So $y(1-x) = 1+x$, and $f^{-1}(x) = [(1+x)/(1-x)]$. Now $f^{-1}(x^{-1}) = [(\frac{1}{x}+1)/(1-\frac{1}{x})] = [(x+1)/(x-1)]$.

6. What is the smallest perfect square that can be expressed as the sum of three distinct nonzero squares?

$7^2 = 49 = 2^2+3^2+6^2$

7. Find a if the roots of x^3+ax^2+bx+c are the reciprocals of the roots of x^3+12x^2+8x+4.

The reciprocals of the roots of x^3+12x^2+8x+4 are roots of $4x^3+8x^2+12x+1 = 4(x^3+2x^2+3x+1/4)$. So a = 2.

8. If sinx = 3cosx, then what is sinx cosx?

$1 = \sin^2 x+\cos^2 x = 10\sin^2 x$. So $\sin x = \pm 1/\sqrt{[10]}$ and sinx cosx = 3/10.

9. Suppose that p and q are positive numbers for which $\log_9(p) = \log_{12}(q) = \log_{16}(p+q)$. What is the value of q/p?

For some real number x, $p = 9^x$, $q = 12^x$ and $p+q = 16^x$. So $q/p = (4/3)^x$. Thus $4^x q/p = 16^{/}3^x = (p+q)/3^x$. So $4^x * 3^x q = p^2+pq$, i.e. $q^2-pq-p^2 = 0$. By the quadratic formula $q = (p\pm p\sqrt{5})/2$. Since p and q are positive, $q/p = [(1+\sqrt{5})/2]$.

10. Let $f_0(x) = [1/(1-x)]$, and $f_n(x) = f_0(f_{n-1}(x))$, for n = 1,2,... Evaluate $f_{2001}(2000)$.

$f_1(x) = f_0(f_0(x)) = [1/(1-[1/(1-x)])] = [1/([(1-x)/(-x)])] = 1-{}^1/_x$.

$f_2(x) = f_0(f_1(x)) = [1/(1-(1-{}^1/_x))] = [1/({}^1/_x)] = x$.

$f_3(x) = f_0(x) = f(6)x = ... = f_{2001}(x)$ so the answer is $[(-1)/2000]$.

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