Mathematica

1. TO BEGIN

Open Mathematica. The first time Mathematica runs, it needs to load the “kernel”. This takes a minute or so.

Have Mathematica compute $2 + 2$.

Type: $2 + 2$ (don’t look for a blinking cursor, just start typing.)

Press: $\text{SHIFT \ ENTER}$ together to evaluate.

2. WHAT HELP IS AVAILABLE

There are numerous manuals available in the computer lab to help you.

- Mathematica: A System for Doing Mathematics by Computer
- Mastering Mathematica
- Mathematica by Example
- The Mathematica Handbook
- Guide to Standard Mathematica Packages
- Programming in Mathematica
- Differential Equations in Mathematica

Also, there is on-line help.

3. IMPORTANT NOTES

The arguments of all Mathematica functions are enclosed in **square brackets**.

The names of **build-in Mathematica** functions begin with **capital letters**.

Examples: \(\text{Sin}[x], \text{Exp}[x], \text{Log}[x]\) (this is the natural logarithm), \(\text{Log}[b, x]\) (this is log to the base \(b\) of \(x\)), \(\text{Sqrt}[x]\).

*Mathematica is case-sensitive* so be careful with upper case and lower case.
4. **TO DEFINE FUNCTIONS**

TO DEFINE: \( f(x) = 3x^2 + 2x - 4 \), \( g(x) = \frac{1 - \sqrt{x}}{1 + \sqrt{x}} \), and \( F(x) = \sin x \).

TYPE:

\[
\begin{align*}
  f[x_] &:= 3x^2 + 2x - 4 \\
  g[x_] &:= (1 - \text{Sqrt}[x])/(1 + x^{1/3}) \\
  F[x_] &:= \text{Sin}[x]
\end{align*}
\]

5. **SOME BASICS**

Since you have already defined \( f(x) \), \( g(x) \), and \( F(x) \), we will work with them.

To find the value of a function,

Type: \( f[2] \) and press \( \text{SE} \).  
\( F[\Pi/2] \) and press \( \text{SE} \).

To multiply and simplify functions,

Type: \( \text{Simplify}[f[x]*g[x]] \) and press \( \text{SE} \).  (This multiplied the two functions and simplified the answer.)

Type: \( \text{Factor}[%] \) and press \( \text{SE} \).  (The \% tells \text{Mathematica} to refer to the last output, so this factors the last output.)

Type: \( \text{Expand}[%] \) and press \( \text{SE} \).  (This expands the last output.)

TRY: \( \text{Expand}[(2x - 3)^{10}] \).

6. **TO GRAPH FUNCTIONS**

Type: \( \text{Plot}[	ext{Sin}[x], \{x, 0, 2\Pi\}] \) and press \( \text{SE} \).

Type: \( \text{Plot}[[\text{Sin}[x], \text{Sin}[2x], \text{Sin}[3x]], \{x, 0, 2\Pi\}] \) and press \( \text{SE} \).

TRY: Graphing \( y = \ln(x) \), \( y = \ln(x - 2) \), and \( y = \ln(x + 5) \) on the same axes.  
Careful! \text{Mathematica} doesn’t understand \( \ln(x) \). See page 1.
Type: Plot3D[Sin[xy], {x,0,3}, {y,0,3}] and press \textbf{SE}

Why is Mathematica saying it can’t do this? Change xy to x*y and try it again.

Now look again at this graph with an expanded domain for x and y.

Type: Plot3D[Sin[x*y], {x,-3,3}, {y,-3,3}] and press \textbf{SE}.

Learn how to enlarge the graph on the screen.

7. \textbf{TO DIFFERENTIATE}

TYPE: \textit{D}[x^4, x] and press \textbf{SE}. (This can also be performed using Mathematica’s palettes.)

TYPE: \textit{D}[x^4, \{x, 2\}] and press \textbf{SE}. (This will give the second derivative of \(x^4\) with respect to \(x\)).

Find the third derivative of \(y = \frac{\ln(\sin x)}{1 + e^x}\). Be very careful with parentheses, square brackets, and how you enter \(e^x\) (use on-line help or review one of the above examples).

\textbf{INDEFINITE INTEGRALS AND MATHEMATICA}

Mathematica has two ways in which you can tell it to perform integration. You can use the “Integrate” command, or you can use the mouse to construct the integral. Both methods are shown in the following example.

Example 1. An integral Mathematica can evaluate: \[
\int \frac{x^2}{x^3 + 1} \, dx.
\]

Method 1: Type in “\textbf{Integrate}[x^2/(x^3 + 1), x]” and press \textbf{SE}.

The output Mathematica gives is \[
\frac{\log \left(\frac{x^3 + 1}{3}\right)}{3}.
\] You must add C. Log is base \(e\).
Method 2: Look over to the right side of the computer screen. Click the mouse on the button containing $\int$ dl. That action should put the integral symbol in your Mathematica workspace.

Type $\frac{x^2}{x^3+1}$ in for the integrand (to get the fraction, press $\text{C}/\text{C}$ or else use the palette, to get up into the exponent area press $\text{C}/\text{C}$).

Type $x$ in the little box after the dl.

Press $\text{Shift}/\text{Enter}$. Again, the output Mathematica gives is $\frac{\log[x^3+1]}{3}$. You must add C. Log is base $e$.

Example 2. An integral Mathematica cannot evaluate: $\int \sin(\sin x) \, dx$.

Type: $\text{Integrate}[\sin[\sin[x]], x]$ or construct it, and press $\text{Shift}/\text{Enter}$.

The output is “Integrate[\sin[\sin[x]], x]”, exactly what you put in.

Whenever Mathematica gives back the same thing you typed in, it means Mathematica could not evaluate it.

Example 3. An integral evaluated to a special function: $\int e^{x^2} \, dx$.

Type: $\text{Integrate}[\text{E}^{x^2}, x]$ and press $\text{Shift}/\text{Enter}$.

The output is $\frac{\text{Sqrt[Pi]} \text{ Erfi}[x]}{2}$. So what the #/#%**!! does Erfi mean?

For help, Type: $\text{?? Erfi}$ and press $\text{Shift}/\text{Enter}$. Mathematica will tell you a little about Erfi. Look in a text on complex analysis for more information. This type of an answer means Mathematica couldn’t come up with an elementary solution.

DEFINITE INTEGRALS AND MATHEMATICA

Example 1. A simple definite integral: $\int_0^x \sin^2 x \, dx$.

Type “$\text{Integrate}[\sin[x]^2, \{x, 0, \text{Pi}\}]$”, or use the palette, and press $\text{Shift}/\text{Enter}$. 
Example 2. An integral with a more complicated solution: \[ \int_0^{2\pi} \sin x^2 \, dx. \]

Type “\texttt{Integrate[Sin[x^2], \{x, 0, 2*Pi\}]}” and press \texttt{Shift+Enter}.

The output is \( \sqrt{\frac{\pi}{2}} \text{FresnelS}[2 \sqrt{2 \pi}] \). This is a solution which is given in terms of the special function FresnelS. Probably not very helpful. See below to get a numerical value for this quantity.


***** VALUABLE INFO *****

Type “\texttt{N[\%]}” and press \texttt{Shift+Enter} to make Mathematica numerically evaluate it’s last output. In Example 2 above, the numerical value of the solution is 0.642138.

Type “\texttt{N[\%\%]}” and press \texttt{Shift+Enter} to make Mathematica evaluate it’s second to the last output. For example, if you did Example 1 and then Example 2, \texttt{N[\%\%]} will evaluate \( \frac{\pi}{2} \), the output from Example 1, and give you 1.5708.

\texttt{N[\%\%\%]} will evaluate the third to the last output, and so on.

Numerical Approximation of Definite Integrals with “NIntegrate”

When \textit{Mathematica} cannot evaluate a definite integral using the command \texttt{Integrate[f(x), \{x, a, b\}]}, you can force \textit{Mathematica} to approximate the integral through numerical methods. \texttt{NIntegrate[f(x), \{x, a, b\}]} tells \textit{Mathematica} to find a numerical approximation.

Example 1. \[ \int_0^{\pi} \sin(\sin x) \, dx. \]

Type “\texttt{Integrate[Sin[Sin[x]], \{x, 0, Pi\}]}” and press \texttt{Shift+Enter}.
The output, after Mathematica loads some special integration packages, is 
\[ \text{Integrate} \left[ \sin(\sin(x^2)), \{x, 0, \pi\} \right] \]. Since this is the same as your input, it means that Mathematica could not evaluate it.

Instead, type "NIntegrate[\sin(\sin(x^2)), \{x, 0, \pi\}]" or N[\%], press \[\text{Shift}\rightarrow\text{Enter}\].

Now the output is 0.694818. This solution is correct to 6 decimal places.

Example 2. \[ \int_{0}^{\frac{\pi}{2}} \frac{x^2 \sin x}{\sin x + \cos x} \, dx \].

Using Integrate or the palette will not result in a solution. Type "NIntegrate[x^2*Sin[x]/(Sin[x]+Cos[x]), \{x, 0, \pi/2\}]", or N[\%], and press \[\text{Shift}\rightarrow\text{Enter}\].

The output is 0.937794.

Example 3. An integral which might not converge: \[ \int_{0}^{\frac{\pi}{2}} \frac{x^2 \sin x}{\sin x + \cos x} \, dx \].

Using NIntegrate for this integral gives –4.24134, but Mathematica also gives a warning that the value is possibly incorrect. If you plot the function, you will see it has a vertical asymptote in the interval and is an improper integral.
Mathcad

1. START-UP AND ARITHMETIC

Start Mathcad. Close the Tip-of-the-Day box (if it pops up) and then close the Mathcad Resource Center.

To use Mathcad, click your mouse anywhere in the work area. The little red crosshair is where your typing will appear.

Type: \(5 + 6 =\)

Click somewhere else.

Type: \(100/5 =\) (use the right arrow to get out of the denominator)

Click somewhere else.

Type: \(27! =\) (answer should be \(1.089 \times 10^{28}\))

If you want to see the result as an integer instead of in scientific notation, go up to Format and select Result. Click on Decimal.

2. A LITTLE ALGEBRA

Click anywhere and type \(x^3 + 3x^2 - 3x - 1\). To get up into the exponent area, press \(<\text{Shift}>6\); to get back down out of the exponent area, press the “right arrow” or the space bar once.

Use your mouse to highlight the entire polynomial, then go up to Symbolics and click Factor. That should factor your polynomial. If you go to Symbolics and Expand, it will multiply out the factors. Factored form should be \((x - 1)(x^2 + 4x + 1)\).

3. FUNCTIONS

To define functions (or anything else), Mathcad does not use the equal sign. Instead, it uses \(:=\). To define \(f(x) = 3x + 6\),

\[
\text{type } f(x) : 3x+6.
\]

You do not need to type the equal sign. As soon as you type the colon, Mathcad knows you also want the equal sign so it automatically includes it.

Now click anywhere BELOW your function and type \(f(5) =\).
Suppose you want to find the value of \( f(x) \) for every integer from 1 to 9. Type 
\[ a : 1 ; 9. \]
The colon puts in \( := \) and the semicolon puts in the .. to indicate that you want a 
list of numbers. Finally, somewhere that is **BELOW** the definitions of \( f(x) \) and \( a \), type 
in \( a = \) and then type in \( f(a) = \). Working with lists in Mathcad is relatively easy.

4. **THE “LIVE” MATHCAD WORK AREA**

Mathcad only knows about definitions in the region **BELOW** where you typed the 
definition. Anywhere **ABOVE** your definition, Mathcad thinks the definition hasn’t been 
provided yet.

Define \( g(x) = \sin \pi x \). <Ctrl><Shift> P will type pi, or you can click \( \pi \) on the 
Greek palette. Did you remember to use \( : \) instead of \( = \) and remember to enclose the 
argument in parentheses? Let’s find the value of \( g(x) \) for \( x = 1, 2, 3, 4, 5, 6. \)

Type \[ a : 1 ; 6. \]

Now click anywhere **ABOVE** your definition of \( g(x) \) and type \( g(a) = \). Mathcad 
tells you \( g \) is not defined. Click and drag the \( g(a) \) box down **BELOW** your definitions so 
Mathcad knows what to use for \( g(x) \) and \( a \).

5. **DIFFERENTIATION**

If the Calculus and Evaluation palettes aren’t open, go to **View, Toolbars**, and 
then click the **Calculus** and **Evaluation** checkboxes.

Define \( f(x) = \frac{x^3 - 4}{x + 2} \). (To get the fraction format, type the / symbol on your 
keyboard BEFORE typing the numerator.)

**BELOW** your definition of \( f(x) \), press the question mark key on your keyboard. 
That will put the Leibnitz notation differentiation operator in your work area. (The 
reason Mathcad uses the question mark key is because it is really the <Shift> / key. 
Leibnitz notation looks like a fraction and / is used for fractions, so <Shift> / is just the 
modified fraction for Leibnitz notation.)

Instead of typing <Shift> /, you can also click on the differentiation operator on 
the Calculus palette. If you hover the cursor above the button on the palette, Mathcad will 
tell you the keyboard short cut: Shift + / means <Shift> /.

Type \( f(x) \) in the differentiation sign and then put an \( x \) in the bottom of the 
operator. (See below.)
Do not use the equal sign to evaluate this. Instead, you must put an “Evaluate Symbolically” arrow, “→”, after the derivative. It is on the evaluation palette. (The keyboard shortcut for it is <Ctrl> <period>. The period key has the > symbol with it, which is kind if like the head of an arrow.) Press <Enter> to get the derivative. You should get:

\[ \frac{d}{dx} f(x) \rightarrow 3 \cdot \frac{x^2}{(x + 2)} - \frac{(3x - 4)}{(x + 2)^2} \]

To find higher derivatives, Mathcad has an additional Leibnitz symbol on the Calculus palette. This symbol provides spaces for you to put in a number to indicate which derivative you want.

Integration, both definite and indefinite, are also accomplished by selecting the appropriate symbol from the Calculus Palette, typing in the function, the variable, and the limits of integration. Then use the “Evaluate Symbolically” arrow and press <Enter>. Integrating \( f(x) \) with respect to \( x \), you should get:

\[ \int f(x) \, dx \rightarrow \frac{1}{3} x^3 - x^2 + 4x - 12 \ln(x + 2) \]

\[ \text{YOU must provide the \(+C\).} \]

6. **GRAPHING**

Define the functions \( f(x) = \sin(x) \) and \( g(x) = \sin(x) \cos(10x) \). Did you remember to use \( : = \) instead of \( = ? \) Next click your mouse BELOW these definitions and then go up to **Insert, Graph, X-Y Plot**. This will place a graphing box in your workspace.

Type \( f(x) \) along the \( y \)-axis and type \( x \) along the \( x \)-axis as shown below.
Click anywhere outside the graphing box, and you should see a red sine wave. Change the graphing window so that it goes from $x = -4$ to $x = 4$ by changing the $-10$ and the $10$.

Next, to plot more than one function on the same graph, put a comma after $f(x)$ and then type $g(x)$ right below the $f(x)$. You should now have a graph like the one below.

To graph polar functions, define $r(t) = 1 - 2 \cos(t)$, insert a Polar Plot instead of an X-Y Plot, and then put $t$ at the bottom and $r(t)$ at the left. You should get a limaçon as follows:

To graph Parametric plots, define $x(t) = \sin(3t)$ and $y(t) = \cos(5t)$. Insert an X-Y Plot and type $y(t)$ on the left and $x(t)$ on the bottom (type them in the placeholders). You should get the Lissajous figure below.
7. THREE DIMENSIONAL GRAPHS

Define \( f(x, y) = x^2 - y^2 \) and \( g(x, y) = x^2 + y^2 \). Click the mouse BELOW the definitions. Go up to Insert, Graph, Surface Plot. That should insert a 3-D axis system in your workspace. Click on the space-holder in the lower left corner and type \( f \). You should get the following.

Now click anywhere on the graph and drag to see the surface rotate.

Click on the \( f \), put a comma and \( g \) after it. You should have two surfaces. Click and drag if you wish. To change the appearance of the surfaces, double click on them and select the “Appearance” tab. Under “Fill Options,” click “Fill Surface” and “Color Map.” Under “Line Options,” increase the weight to 1.1 and click “Apply.” Do the same under the “Plot 2” tab, then click “OK.” Click and drag to rotate the surfaces.

8. SOLVING EQUATIONS

To let Mathcad know that you are working with an equation that you will want it to solve, you have to use a special equal sign. Use <\( \text{Ctrl} \)= instead of just =. For example, to solve \( 0 = x^2 - 7x + 12 \), type it in using <\( \text{Ctrl} \)=, then use your mouse or
keyboard arrows to get the editing lines on the variable (either the \( x \) or \( x^2 \)). To solve, go up to **Symbolsics, Variable, Solve**.

9. **PRACTICE PROBLEMS FOR MATHCAD**

1. **Factor:**
   a) \( 6x^3 + 29x^2 - 17x - 60 \)   
   b) \( x^5 - 3x^4 - 23x^3 + 51x^2 + 94x - 120 \)

2. **Graph:**
   a) \( f(x) = e^x \sin(x) \)
   b) \( x(t) = t + 1, y(t) = t^3 - 3t \) (change graphing window to \(-1 \leq x \leq 5, -3 \leq y \leq 3\))
   c) \( r(\theta) = \theta + \theta \sin(\theta) \)
   d) \( f(x, y) = \sin(x) + \cos(y) \)

3. Let \( f(x) = e^{2x} \sin x \).
   a) find the third derivative of \( f(x) \). solution: \( 2 \cdot \exp(2 \cdot x) \cdot \sin(x) + 11 \cdot \exp(2 \cdot x) \cdot \cos(x) \)
   b) find \( \int_{0}^{\pi/4} f(x) \, dx \). solution: \( \frac{1}{10} \cdot \exp\left(\frac{1}{2} \cdot \pi\right) \sqrt{2} + \frac{1}{5} + C \)