Calculus 166 Introduction to X(PLORE) and Mathematica

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X(PLORE)

Turn on the computer and turn on the monitor. Use the mouse to double click on the X(PLORE) icon. X(PLORE) has on-line help. X(PLORE) is not mouse interactive.

I. Graphing Functions

To make X(PLORE) graph a function, you only need to tell it three things: first tell it what function you want graphed, second tell it where on the x-y plane you want it graphed, and third tell it to draw the graph.

EXAMPLE 1. Since you've taken a trigonometry class and already know everything there is to know about graphing trig functions, let's start by graphing $f(x) = 2 \sin(3x)$:

- Type "$f(x) = 2 \sin(3x)$" and press $\text{Enter}$. (don't type either " mark, but do type all parentheses)
- Type "window(–500,500, –500,500)" and press $\text{Enter}$.
- Type "graph(f(x),x)" and press $\text{Enter}$. Ta-dah!! A perfectly lousy graph.

This graph is lousy due to the 500's in the window command. If you had to graph "$f(x) = 2 \sin(3x)$" on a trig test, you'd probably mark the x-axis from $-2\pi$ to $2\pi$ and the y-axis from –4 to 4. To make X(PLORE) do the same, use "window($-2\pi,2\pi,-4,4$)".

To reset the window, first press $\text{Esc}$ or $\text{Enter}$ to return to the input screen. Next, change the window command that's already there. Use the arrow keys to get the cursor on one of the 500's and then use $\text{Del}$ or the backspace $\text{C}$ to get rid of all the 500's. Replace them with $-2\pi, 2\pi, -4, 4$. ****Type $\text{Alt}[p]$ to get $\pi$. ****Press $\text{Enter}$ while the cursor is on the window line, and then press $\text{Enter}$ again when the cursor is on the graph line. There! A much better graph.
Determining a good window setting requires a little trial and error and a lot of thinking. The more thinking you do, the less trial and error it will take. The computer does the busy work of drawing graphs, but you still have to provide the brain power.

It's simple to graph more functions on the same axis system. For example, go back to the input screen and change the function line to \( f(x) = 4 \sin(6x) \). Just change the numbers, don't retype the whole thing. Press [Enter] and then move to the graph line and press [Enter]. Don't press [Enter] while on the window line this time. If you want to erase everything that's been graphed, type "erase" on its own line and then press [Enter].

Erase. Graph \( f(x) = \sin^2 x^2 \), \( g(x) = \sin^2 x \), and \( h(x) = \sin x^2 \) on the same axis. 

Hint: \( f(x) \) should be entered as \( f(x) = \sin(x^2)^2 \).

Erase. Graph \( f(x) = 3^x \) and \( g(x) = \frac{3^x}{x} \) on the same axis.

Erase. Graph \( f(x) = \frac{x + 3}{2x + 5} \) and \( g(x) = \frac{x + 3}{2x} + 5 \) on the same axis.

II. Differentiation

To have X(plorer) find a function’s derivative, first define the function and then differentiate it. For example, enter the following two lines,

\[
\begin{align*}
f(x) &= \sin^2(3x) \\
d\text{iff}(f(x), x)
\end{align*}
\]

to find the derivative of \( f(x) \). Entering

\[
\text{diff}(f(x), x = \pi/6)
\]

will evaluate the derivative at \( x = \pi/6 \). Entering

\[
\text{diff}(g(f(x), x = \pi/6)
\]

will graph \( f(x) \) and the line tangent to \( f(x) \) at \( x = \pi/6 \).

III. Polar Graphs

To graph a polar function such as \( r(\theta) = \cos(3\theta) \), enter
polarg(cos(3t), t)

after specifying an appropriate graphing window. This should be a 3-petaled rose. Try graphing \( r(\theta) = 1 - 2\cos(\theta) \), \( r(\theta) = 3\sin(\theta) \), and \( r(\theta) = 0.8\csc(\theta) \).

IV. Parametric Graphs

To graph a parametrically defined curve such as \( x(t) = 3\sin(3t) \), \( y(t) = 2\sin(5t) \) for \( 0 \leq t \leq 2\pi \), enter

\[
\text{paramg}(3\sin(3t), 2\cos(5t), t = 0 \text{ to } 2\pi).
\]

The window for this graph should be at least \(-3 \leq x \leq 3\) and \(-2 \leq y \leq 2\). The graph should be a Lissajous figure. Try graphing some ellipses and circles parametrically.

V. Space Curves

To graph a curve in three-dimensional space, first define three parametric functions:

\[
\begin{align*}
&x(t) = \cos(t) \\
y(t) = \sin(t) \\
z(t) = t
\end{align*}
\]

and then graph the curve by entering

\[
\text{curve3d}(x(t), y(t), z(t), t = 0 \text{ to } 4\pi \text{ step 100}).
\]

This should be a circular helix going up around the \( z \)-axis. Hold down the \( x \), \( y \), or \( z \) key to watch the curve rotate. For even more exciting graphs, try graphing and rotating

\[
x(t) = t^*\cos(48\pi^*t), \quad y(t) = t^*\sin(48\pi^*t), \quad z(t) = t^*2 \cos(96\pi^*t), \quad \text{with } t = 0 \text{ to } 0.2 \text{ step 200};
\]

or

\[
x(t) = t^*\cos(40\pi^*t), \quad y(t) = t^*\sin(40\pi^*t), \quad z(t) = -t, \quad \text{with } t = 0 \text{ to } 0.5 \text{ step 175}.
\]

VI. Three-Dimensional Graphs of Surfaces

To graph functions of two variables, first define the function and then use the “graph3d” command. To graph a hyperbolic paraboloid, for example, enter

\[
f(x, y) = x^2 - y^2
\]
graph3d(f(x, y); x = -1 to 1, y = -1 to 1).

Rotate it so you can see its saddle shape more clearly. Try changing it to 
\( f(x, y) = x^2 + y^2 \) and rotate.

To graph parametric surfaces, first enter three parametric equations and then use the “paramg3d” command. For example, enter

\[
\begin{align*}
    x(r, t) &= r \cos(t) \\
    y(r, t) &= r \sin(t) \\
    z(r, t) &= \sin(r)/r
\end{align*}
\]

paramg3d(x(r, t), y(r, t), z(r, t); t = 0 to 2\(\pi\) step 15; r = 0 to 9).

Olé!!

VII. Practice Graphing and Investigating Graphs

1. Graph the following three functions on the same axes. What are the differences in the graphs and why are they different?

\[
\begin{align*}
    y &= 3^{-x}, \\
    y &= -3^x, \\
    y &= -3^{-x}.
\end{align*}
\]

2. Graph the following and compare them with \( y = 2^x \). Discuss the differences.

\[
\begin{align*}
    y &= 2^x + 1, \\
    y &= 2^{x+1}, \\
    y &= 2^{|x|}.
\end{align*}
\]

3. Find the following limits:

\[ \lim_{x \to -\infty} 3^{\frac{1}{x}} \]
\[ \lim_{x \to -\infty} 3^{\frac{5}{x}} \]
\[ \lim_{x \to -\infty} 3^{\frac{1}{x}} \]
\[ \lim_{x \to -\infty} 3^{\frac{4}{x}} \]

Now graph the function \( y = 3^{\frac{1}{x}} \). Are the limits you found correct?

4. Graph \( y = x^3 \) along with its inverse and the line \( y = x \) all on the same axes. What is the relationship between these graphs?

5. Graph the following three functions on the same axes. What horizontal or vertical shifts are occurring?

\[
\begin{align*}
    y &= \log_3 x, \\
    y &= \log_3 (x - 1), \\
    y &= \log_3 (x - 1) + 1.
\end{align*}
\]

6. Graph \( y = -\ln x \). What is its inverse function?
7. Graph the following five functions. What are the differences in these graphs?

a) \( y = -\ln(x) \)  
b) \( y = \ln(-x) \)  
c) \( y = -\ln(-x) \)  
d) \( y = \ln|x| \)  
e) \( y = \ln\left(\frac{1}{x}\right) \)

**Differentiation**

8. Differentiate the following two functions by hand.

a) \( y = \sqrt[3]{2x + \sin x} \)

b) \( y = \frac{\sin^2 x - 1}{\cos x} \)

Check your answers using X(PLORE).

**Mathematica**

1. **TO BEGIN**

   Open *Mathematica*. The first time *Mathematica* runs, it needs to load the “kernel”. This takes a minute or so.

   Have *Mathematica* compute 2 + 2.

   Type: 2 + 2 (don’t look for a blinking cursor, just start typing.)

   Press: `SHIFT` `ENTER` together to evaluate.

2. **WHAT HELP IS AVAILABLE**

   There are numerous manuals available in the computer lab to help you.

   *Mathematica*: A System for Doing Mathematics by Computer
   Mastering *Mathematica*
   *Mathematica* by Example
   The *Mathematica* Handbook
   Guide to Standard *Mathematica* Packages
   Programming in *Mathematica*
   Differential Equations in *Mathematica*

   Also, there is on-line help.
3. **IMPORTANT NOTES**

   The arguments of all *Mathematica* functions are enclosed in **square brackets**. The names of **build-in Mathematica** functions begin with **capital letters**.

   Examples: Sin[x], Exp[x], Log[x] (this is the natural logarithm), Log[b, x] (this is log to the base b of x), Sqrt[x].

   *Mathematica* is **case-sensitive** so be careful with upper case and lower case.

4. **TO DEFINE FUNCTIONS**

   TO DEFINE: 
   \[ f(x) = 3x^2 + 2x - 4, \quad g(x) = \frac{1 - \sqrt{x}}{1 + \sqrt[x]} , \quad \text{and} \quad F(x) = \sin x , \]

   TYPE: 
   \[ f[x_] := 3x^2 + 2x - 4 \quad \text{(notice it is } x \_ \text{ not just } x) \]
   \[ g[x_] := (1 - \mathrm{Sqrt}[x])/(1 + x^{1/3}) \]
   \[ F[x_] := \sin[x] \]

5. **SOME BASICS**

   Since you have already defined \( f(x) \), \( g(x) \), and \( F(x) \), we will work with them.

   To find the value of a function,
   - Type: \( F[\pi/2] \) and press `Shift`+`Enter`.

   To multiply and simplify functions,
   - Type: \( \text{Simplify}[f[x]*g[x]] \) and press `Shift`+`Enter`. (This multiplied the two functions and simplified the answer.)
   - Type: \( \text{Factor}[\%] \) and press `Shift`+`Enter`. (The % tells *Mathematica* to refer to the last output, so this factors the last output.)
   - Type: \( \text{Expand}[\%] \) and press `Shift`+`Enter`. (This expands the last output.)

   TRY: \( \text{Expand}[(2x - 3)^{10}] \).
6. **TO GRAPH FUNCTIONS**

Type:  \( \text{Plot}[\sin(x), \{x, 0, 2\pi\}] \) and press \( \text{SHIFT ENTER} \).

Type:  \( \text{Plot}[[\sin(x), \sin(2x), \sin(3x)], \{x, 0, 2\pi\}] \) and press \( \text{SHIFT ENTER} \).

TRY:  Graphing \( y = \ln(x) \), \( y = \ln(x - 2) \), and \( y = \ln(x + 5) \) on the same axes.

Type:  \( \text{Plot3D}[[\sin(xy), \{x,0,3\}, \{y,0,3\}] \) and press \( \text{SHIFT ENTER} \).

Why is Mathematica saying it can’t do this? Change \( xy \) to \( x*y \) and try it again.

Now look again at this graph with an expanded domain for \( x \) and \( y \).

Type:  \( \text{Plot3D}[[\sin(x*y), \{x,-3,3\}, \{y,-3,3\}] \) and press \( \text{SHIFT ENTER} \).

Learn how to enlarge the graph on the screen.

7. **TO DIFFERENTIATE**

TYPE:  \( D[x^4, x] \) and press \( \text{SHIFT ENTER} \).

TYPE:  \( D[x^4, \{x, 2\}] \) and press \( \text{SHIFT ENTER} \). (This will give the second derivative of \( x^4 \) with respect to \( x \).)

Find the third derivative of \( y = \frac{\ln(\sin x)}{1 + e^x} \). Be very careful with parentheses, square brackets, and how you enter \( e^x \) (use on-line help or review one of the above examples).

8. **TO SOLVE EQUATIONS**

To solve the equation \( x = x^3 - x^2 + 1 \),

TYPE:  \( \text{Solve}[x == x^3 - x^2 + 1, \ x] \) and press \( \text{SHIFT ENTER} \).

The output, \( \{x \to -1\}, \{x \to 1\}, \{x \to 1\} \), means the solutions are \( x = -1 \) and \( x = 1 \), with \( x = 1 \) being a double root.

To solve \( x^2 = 3x^2 - 3x + 4 \),

TYPE:  \( \text{Solve}[x^2 == 3x^2 - 3x + 4, \ x] \) and press \( \text{SHIFT ENTER} \).

The solutions are \( x = \frac{3 \pm i\sqrt{23}}{4} \).
9. **To Find Partial Fractions**

To find the partial fractions of
\[
\frac{2 - 2x}{x^2 + 6x + 5},
\]

**TYPE:** \[\text{Apart}\[(2 - 2x)/(x + 6 + 5)]\] and press \[\text{SHIFT} \text{ ENTER}\].

The output is \(\frac{1}{1 + x} - \frac{3}{5 + x}\). This means that
\[
\frac{1}{1 + x} - \frac{3}{5 + x} = \frac{2 - 2x}{x^2 + 6x + 5}.
\]

To find the partial fractions of
\[
\frac{x^2 - 3x + 4}{x^3 - 2x^2 + x - 2},
\]

**TYPE:** \[\text{Apart}\[(x^2 - 3x + 4)/(x^3 - 2x^2 + x - 2)]\] and press \[\text{SHIFT} \text{ ENTER}\].

The output is \(\frac{2}{5(2 + x)} + \frac{3(3 + x)}{5(1 + x^2)}\).

Practice Problems for Mathematica:

1. Have Mathematica evaluate \(2^{135}\).
   Ans: \(43,556,142,965,880,123,323,311,949,751,266,331,066,368\).

2. Define the functions \(f(x) = 2.7^{-x^{1/5}}\) and \(g(x) = \sin 7x\) in Mathematica.
   Ans: Re-read parts 3 & 4 on page 6.

3. Have Mathematica plot \(f(x), f(x) \cdot g(x)\) and \(f(x) \cdot g^2(x)\) from \(-2\pi\) to \(2\pi\).
   Ans:
   ![Graphs](image)

4. Have Mathematica differentiate \(x^2 \sin^3 x^4\).
   Ans: You should find the derivative to see if Mathematica got it right.

5. Have Mathematica solve \(x^4 + x^2 + 1 = x^3 + x\).